VIBRATION OF ROTATING ORTHOTROPIC DISCS

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CERTIFICATE

This is to certify that the work presented in this thesis entitled '' Vibration of Rotating Orthotropic Discs'' by Shri Avinash Rajguru has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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AVINASH RAJGURU

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NOMENCLATURE

a	=	clamping radius
b	=	outer radius
r	=	disc radial coordinate
h_o	=	thickness at periphery
β	=	thickness parameter
h	=	$h_o(r/b)^{-\beta}$ = disc thickness at radius r
H	=	$\frac{1}{2}$ peripheral disc thickness
ρ_o	=	density at periphery
m	=	density parameter
ρ	=	$\rho_o(r/b)^m$ = disc density at radius r
w	=	transverse displacement
u	=	radial displacement
V_B	=	potential energy due to bending
V_{Bmax}	=	maximum potential energy due to bending
E_r	=	modulus of elasticity in radial direction
E_θ	=	modulus of elasticity in tangential direction
ν_r	=	strain in tangential direction due to unit strain in radial direction
ν_θ	=	strain in radial direction due to unit strain in tangential direction
D_r	=	$\dfrac{E_r h^3}{12(1-\nu_r\nu_\theta)}$, flexural rigidity in radial direction
$\bar{D}_r$	=	$\dfrac{E_r h_o^3}{12(1-\nu_r\nu_\theta)}$

$D_\theta = \dfrac{E_\theta h^3}{12(1-\nu_r\nu_\theta)}$, flexural rigidity in tangential direction

$\bar{D}_\theta = \dfrac{E_\theta h_o^3}{12(1-\nu_r\nu_\theta)}$,

$D_1 = \nu_r D_\theta = \nu_\theta D_r$

$\bar{D}_1 = \nu_r \bar{D}_\theta = \nu_\theta \bar{D}_r$

$G = \dfrac{\sqrt{(E_r E_\theta)}}{2(1-\sqrt{(\nu_r\nu_\theta)})}$

$D_{r\theta} = Gh^3/12$, shear rigidity

$\bar{D}_{r\theta} = Gh_o^3/12$

σ_r = radial stress

σ_θ = tangential stress

Ω = rotational velocity

$\bar{\Omega} = \sqrt{(\rho_o\Omega^2 h_o b^4/8\bar{D}_r)}$, non-dimensional speed

$\bar{\sigma}_r = \sigma_r/\rho_o\,\Omega^2 b^2$, non-dimensional radial stress

$\bar{\sigma}_\theta = \sigma_\theta/\rho_o\,\Omega^2 b^2$, non-dimensional tangential stress

$\lambda^2 = E_\theta/E_r$, Rigidity parameter

$\alpha_{1,2} = -\tfrac{1}{2}\beta \pm\sqrt{(\tfrac{1}{4}\beta^2 + \lambda^2 + \nu_\theta\beta)}$

ε_r = strain in radial direction

ε_θ = strain in tangential direction

V_R = potential energy due to rotation

V_{Rmax} = maximum potential energy due to rotation

T = kinetic energy of disc

T_{max} = maximum kinetic energy of disc { Rot

W_1 = disc transverse displacement surface

W = disc transverse displacement radial distribution

ω_n = disc natural frequency of transverse vibration

$\bar{\omega}_n$ = $\sqrt{(\omega_n^2 \rho_o h_o b^4)/\bar{D}_r}$, non-dimensional frequency

n = number of nodal diameters

s = number of nodal circles

θ = disc angular coordinate

a_i = mode shape constants, $i = 0,1,2,3,4,5$.

M = constant

M' = constant

ABSTRACT

In the present analysis approximate natural frequencies of rotating orthotropic discs have been obtained. Ritz method is used assuming approximate mode shapes. Radial variation of thickness and density are also considered. Effect of clamping radius, variation of thickness, variation of density and variation of rigidity parameter (ratio of the Young's modulus in tangential direction to the Young's modulus in radial direction) are studied. Special cases of rotating isotropic discs and stationary orthotropic discs have been compared with previous work and found that the frequencies obtained from the present analysis are well in agreement with them.

It is found that frequency of rotating orthotropic discs increases with increase in clamping radius. As the disc becomes thicker at the hub or heavier towards the periphery frequency increases. The effect of increasing the rigidity parameter is to increase the natural frequency.

CHAPTER 1

INTRODUCTION

1.1 General Introduction:

Vibration characteristics of circular spinning discs are of interest in design, since circular discs are commonly used structural elements as in turbines, flywheels, saw blades, optical and radar reflectors for space vehicles, solar sails etc. It has been observed that while a turbine disc or a flywheel is kept spinning it exhibits transverse vibration. Research work so far made in this connection, have shown that such vibrations of disc are largely responsible for the disc failure, fatigue of the metal and gradual development of cracks on the disc or wheel. Such vibrations at certain speeds, become very pronounced and thereby cause considerable additional bending stress which may result in fatigue of the metal and gradual development of cracks. Sometimes vibrating discs may even come in contact with adjacent parts and lead to the failure of the whole structure. The necessity for the design of efficient, lightweight structures has led to the use of special alloys and composite materials which may have directional properties. Similarly, design may demand for a particular type of use or to improve mechanical strength and properties of circular plates in desired directions, incorporating stiffness radially or

circumferencially or in both. Stiffened plates can be idealized as orthotropic plates. Thus a study of the vibration characteristics of a rotating isotropic and orthotropic plates becomes essential.

1.2 A Brief Survey of Literature:

1.2.1 Isotropic Discs.

Early investigators found that the solution of the general case posed considerable mathematical complexity and elected to study separately the cases in which the rotational effects were negligible and the cases in which bending stiffness effects were overshadowed by rotational effects. The case in which rotational effects are of primary importance led directly to the study of spinning membrane discs. Lamb and Southwell [1] discuss the problem of vibration of a spinning circular membrane - a disc with negligible bending stiffness - and it is shown that the mode shapes are Jacobi polynomials. Southwell [2] extends the analysis to the study of the transverse vibrations of a spinning membrane clamped to a rigid hub in such a manner that radial but not transverse displacement may occur. The solution to this problem necessitates finding of hypergeometric functions. Simmonds[3] considers the same problem and points out that Southwell's paper contains an error since it overlooks the facts that the hypergeometric equation cannot have two independent solutions involving only terms containing the hypergeometric series $F(\alpha,\beta,\gamma,x)$, if γ is allowed to take on negative integral value

Bulkeley and Savage [4] have studied the centrally clamped membrane for the case of axisymmetric vibrations. Their results are for various degrees of partial clamping, including the fully clamped configuration. Transverse vibrations of centrally clamped membrane with Coulomb friction at the disc collar interface, are shown by a scale factor, to be equivalent to those of a membrane fully built into a hub, or clamped between frictionless collars. Simmonds [5] has studied the fully clamped case for axisymmetric vibrations, in which equation of motion reduces to the hypergeometric differential equation.

Mote [6] has employed a Rayleigh-Ritz technique to study the approximate vibration characteristics of centrally clamped variable thickness discs. He investigated the natural frequencies of transverse vibrations taking into consideration rotational and thermal in-plane stresses as well as purposely induced initial stresses. He found that initial stresses can significantly raise the minimum disc natural frequency throughout a prescribed rotational and thermal environment. The fundamental mode of disc vibration is one of zero nodal circles and either zero, one or two nodal diameters, depending upon the disc geometry and the rotational-thermal environment. Mote further extended his work [7] to incorporate the transient temperature distribution, and associated thermal stresses. An analytical rule-of-thumb

involving the calculation and test of a non-dimensional number is proposed for predicting conditions when the transient regime may be significantly more severe than the steady state. He has also investigated [8] the possibility of increasing the fundamental frequency by purposely inducing thermal membrane stresses in centrally clamped, peripherilly free rotating annuli. Practical application of both these methods [6,8] is limited to the cases where minimization of disc thickness is of paramount importance. Optimization of the fundamental frequency and limitations of the above methods are also discussed [8,9].

Eversman [10,11] has given the exact method for determination of natural frequencies of transverse vibrations of a clamped spinning membrane disc. Later on, Eversman and Dodson [12] treated the problem of transverse vibrations of a spinning centrally clamped circular disc, for which Mote [6] has given an approximate solution. Their approach is exact to within the accuracy of the numerical computations, requires no appropriate choices of the modal functions, and is free of usual uncertainities of approximate methods. Barasch and Chen [13] have also investigated the above case of rotating disc, the equation of motion is solved by reducing the fourth order equations of motion to a set of four first order equations subject to arbitrary initial conditions. A modified Adams' method is used to numerically integrate the system of differential equations.

1.2.2 Experimental Analysis.

Not much experimental analysis on the rotating discs is reported in the literature. Campbell [14] investigated experimentally various forms of vibrations and waves which may exist in steam turbine disc wheels and proposed procedures necessary for the protection of steam turbine bucket wheels from transverse vibrations. Krauter and Bulkeley [15] have experimentally observed the free transverse vibration of a centrally clamped spinning membrane discs and compared axisymmetric and asymmetric frequencies with theoretical predictions [3].

1.2.3 Orthotropic Discs.

Recently use of composite materials and stifffining of circular plates has led to the investigation of the characteristics of orthotropic circular plates. Circular plates with different elastic properties in radial and circumferential directions have been called by various names as cylindrically aeolotropic, orthotropic, polar orthotropic or anisotropic by different investigators. Minkarah and Hoppmann [16] consider flexural vibration of cylindrically aeolotropic circular plates assuming symmetric vibrations. The theory is exemplified by an analysis of the vibration of a circular plate with circular stiffness for which the effective elastic compliances had been determined in previous investigations. They also

discuss the approximation of equivalent homogeneous uniformly thick aeolotropic plates for stiffened plates. Dzialo and Hoppmann [17] extended the above problem to consider the effect of rotary inertia. They found that the frequencies are higher than the actual if rotary inertia is neglected. Pandalai and Patel [18] have also considered the above problem and determined natural frequencies of orthotropic circular plates.

Ghosh [19] investigated vibration of a rotating aeolotropic elastic circular disc of uniform thickness. He has given a mathematical analysis for determining frequencies and modes of vibration. He further extends his investigation [20] to take into consideration disc of variable thickness. In his analysis, he neglected the effect of bending stiffness.

Ramaiah and Vijaykumar [21,22] have used Rayleigh-Ritz method with simple polynomials as admissible functions for obtaining natural frequencies of transversely vibrating polar orthotropic annular plates. Estimates of natural frequencies corresponding to modes with one as well as two nodal diameters are obtained for the nine combinations of clamped, simply supported and free edge conditions and for various values of rigidity ratio and hole sizes. Based on the variation of eigenvalue parameter with rigidity ratio, the frequencies of these modes as well as the axisymmetric modes have been

expressed by means of simple formulae in terms of rigidity ratio and corresponding modes in the isotropic case.

1.3 Present Work :

The present work deals with the determinations of natural frequencies of orthotropic rotating discs. The effect of clamping radius, thickness variation and the density variation are determined. The effect of rigidity parameter (The ratio of Young's modulus in θ direction to the Young's modulus in r direction) on the Natural frequency is also analysed. Approximate natural frequencies are obtained by using Ritz-method. Specialising the above results to the case of an isotropic disc, comparison is made to existing work.

CHAPTER 2

PROBLEM FORMULATION AND SOLUTION

The necessary formulation and solution for the calculation of natural frequencies of an orthotropic circular disc of variable thickness and density, rotating at a constant angular velocity is presented in this section. Expressions for the potential energy in bending and potential energy due to rotation (that is, due to the inplane radial and circumferential stresses) and the kinetic energy of the disc are given. In-plane stresses for the above orthotropic, rotating disc is presented. Assuming that the disc is undergoing harmonic oscillations, suitable mode shapes have been assumed and expressions for maximum potential energy and kinetic energy are obtained. Ritz method is used to obtain the natural frequencies. Frequencies for different plate geometries, variation of density and thickness and orthotropy in two directions are obtained.

2.1 Plate Geometry:

Consider a thin orthotropic elastic disc of outer radius 'b', clamped at radius 'a', rotating uniformly with angular velocity Ω about the normal axis passing through the centre and perpendicular to the centre plane of symmetry as shown in Fig. 1.

Variation in the thickness of the disc has been chosen in the form

$$h = h_o\left(\frac{r}{b}\right)^{-\beta} \qquad (2.1)$$

where, h = thickness at radius r

h_o = thickness at periphery

b = outer radius

β = thickness parameter.

Similarly variation in the density of the disc is assumed to be of the form of

$$\rho = \rho_o\left(\frac{r}{b}\right)^{m} \qquad (2.2)$$

where, ρ = density at radius r

ρ_o = density at periphery

m = density parameter.

2.2 Boundary Conditions:

The geometric displacement boundary conditions at the clamping radius $r = a$ are

(i) Transverse displacement 'w' is zero, i.e.

$$w = o \quad \text{at} \quad r = a \qquad (2.3)$$

(ii) Slope is zero, i.e.

$$\frac{dw}{dr} = o \quad \text{at} \quad r = a \qquad (2.4)$$

The following boundary conditions are used to determine the radial and tangential stresses due to rotation.

(iii) Radial displacement 'u' is zero, i.e.

$$u = o \quad \text{at } r = a \tag{2.5}$$

and

(iv) Radial stress is zero at the periphery, i.e.

$$\sigma_r = o \quad \text{at} \quad r = b \tag{2.6}$$

2.3 Assumptions:

This analysis is based on the following assumptions:

(i) Conditions of plane stress.

(ii) The thickness and density distribution is symmetrical with respect to both the axis of the disc and the midplane of the disc.

(iii) The influence of shear stress and rotary inertia is negligible.

(iv) The disc material is polarly orthotropic.

(v) The stress-strain relations follow a generalised Hooke's law with the principal axis of orthotropy coinciding with the principal axis of the disc.

2.4 Potential Energy due to Bending:

The potential energy for the plate in bending can be written as [22],

$$V_B = \frac{1}{2}\int_0^{2\pi}\int_a^b [D_r(\frac{\partial^2 w}{\partial r^2})^2 + 2\frac{D_1}{r}\frac{\partial^2 w}{\partial r^2}(\frac{\partial w}{\partial r} + \frac{1}{r}\frac{\partial^2 w}{\partial \theta^2})$$

$$+ \frac{D_\theta}{r^2}(\frac{\partial w}{\partial r} + \frac{1}{r}\frac{\partial^2 w}{\partial \theta^2})^2 + 2D_{r\theta}\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial w}{\partial \theta})^2]rdrd\theta \tag{2.7}$$

where

$$D_r = \frac{E_r h^3}{12(1-\nu_r\nu_\theta)}$$, flexural rigidity in radial direction (2.8)

$$D_\theta = \frac{E_\theta h^3}{12(1-\nu_r\nu_\theta)}$$, flexural rigidity in tangential direction (2.9)

$$D_1 = \nu_r D_\theta = \nu_\theta D_r \tag{2.10}$$

$$D_{r\theta} = Gh^3/12$$, shear rigidity (2.11)

$$G = \frac{\sqrt{(E_r E_\theta)}}{2[1-\sqrt{(\nu_r\nu_\theta)}]} \tag{2.12}$$

E_r = Young's Modulus in radial direction

E_θ = Young's Modulus in tangential direction

ν_r = strain in tangential direction due to the unit strain in radial direction

ν_θ = strain in radial direction due to unit strain in tangential direction.

2.5 Potential Energy due to Rotation:

Potential energy due to rotation can be written as [23],

$$V_R = \frac{1}{2} \int_0^{2\pi} \int_a^b \left[h\sigma_r\left(\frac{\partial^2 w}{\partial r}\right)^2 + h\sigma_\theta\left(\frac{\partial^2 w}{r\partial\theta}\right)^2\right] r\,dr\,d\theta \qquad (2.13)$$

where, σ_r = radial stress

and σ_θ = circumferential or tangential stress.

2.6 Kinetic Energy:

Kinetic energy of the disc can be written as [23],

$$T = \frac{1}{2} \int_0^{2\pi} \int_a^b \rho\, h\left(\frac{\partial w}{\partial t}\right)^2 r\,dr\,d\theta \qquad (2.14)$$

2.7 Expressions for in-plane Stresses:

The expressions for the in-plane stresses of a rotating orthotropic disc with variable thickness and variable density has been obtained by Reddy and Shrinath [24]. The method is given here briefly;

For a spinning disc rotating at an angular velocity Ω, equation of equilibrium to be satisfied by the stresses is

$$\frac{d}{dr}(hr\sigma_r) - h\sigma_\theta + h\rho\,\Omega^2 r^2 = 0 \qquad (2.15)$$

The strain displacement relations are

$$\varepsilon_r = \frac{du}{dr}, \qquad \varepsilon_\theta = \frac{u}{r} \qquad (2.16)$$

and hence the strain compatibility equation is

$$r\frac{d\varepsilon_\theta}{dr} + (\varepsilon_r - \varepsilon_\theta) = 0 \qquad (2.17)$$

The stress-strain relations for the polarly orthotropic material are

$$\varepsilon_r = \frac{\sigma_r}{E_r} - \frac{\nu_\theta}{E_\theta}\sigma_\theta \qquad (2.18)$$

$$\varepsilon_\theta = \frac{\sigma_\theta}{E_\theta} - \frac{\nu_r}{E_r}\sigma_r \qquad (2.19)$$

With Maxwell's relation

$$\frac{\nu_\theta}{E_\theta} = \frac{\nu_r}{E_r} \qquad (2.20)$$

The equation of equilibrium can be satisfied by a function F such that

$$F = h\,\sigma_r \quad \text{and} \quad h\,\sigma_\theta = \frac{dF}{dr} + h\rho\,\Omega^2 r^2 \qquad (2.21)$$

Using Eqs.(2.18 - 2.20) alongwith the relations (2.1) and (2.2),

$$h = h_o(r/b)^{-\beta}$$

and $$\rho = \rho_o(r/b)^m$$

in Eq.(2.17) we get,

$$\frac{d^2F}{dr^2} + \frac{K_1}{r}\frac{dF}{dr} - \frac{K_2}{r^2}F = K_3 h_o\,\rho_o\,\Omega^2\,r^{m-\beta+1}\,b^{m-\beta} \qquad (2.22)$$

where, $K_1 = 1+\beta$

$K_2 = \lambda^2 + \nu_\theta\beta$

$K_3 = -(3+m+\nu_\theta)$

Using Eqs.(2.15 - 2.22) radial and circumferential stresses have been obtained as discussed by Reddy and Shrinath [24]. For the disc clamped rigidly at the hub, the boundary conditions are;

(i) displacement is zero at clamping radius, i.e.

$$u = o \quad \text{at} \quad r = a$$

and

(ii) radial stress is zero at the periphery, i.e.

$$\sigma_r = o \quad \text{at} \quad r = b$$

For the above conditions stresses are given by

$$\sigma_r = \rho_o \Omega^2 b^2 C_3 [\frac{\beta_1}{\beta_3}(r/a)^{\alpha_1+\beta-1} - \frac{\beta_2}{\beta_3}(r/a)^{\alpha_2+\beta-1} + (r/b)^{2+m}] \tag{2.23}$$

$$\sigma_\theta = \rho_o \Omega^2 b^2 [C_3 \{ \frac{\alpha_1\beta_1}{\beta_3}(r/a)^{\alpha_1+\beta-1} - \frac{\alpha_2\beta_2}{\beta_3}(r/a)^{\alpha_2+\beta-1} + (3+m-\beta)(r/b)^{2+m} \} + (r/b)^{2+m}] \tag{2.24}$$

where,

$$\alpha_1 = -\tfrac{1}{2}\beta + \sqrt{(\tfrac{1}{4}\beta^2 + \lambda^2 + \nu_\theta \beta)} \tag{2.25}$$

$$\alpha_2 = -\tfrac{1}{2}\beta - \sqrt{(\tfrac{1}{4}\beta^2 + \lambda^2 + \nu_\theta \beta)} \tag{2.26}$$

$$\lambda^2 = E_\theta / E_r \tag{2.27}$$

$$C_3 = \frac{-(3 + m + \nu_\theta)}{(9-\lambda^2) - \beta(3+m+\nu_\theta) + m(m+6)} \tag{2.28}$$

$$\beta_1 = \left(\frac{\nu_\theta^2 - \lambda^2}{3+m+\nu_\theta}\right)(a/b)^{3+m-\alpha_2} + (\alpha_2 - \nu_\theta)(a/b)^{\beta} \qquad (2.29)$$

$$\beta_2 = \left(\frac{\nu_\theta - \lambda^2}{3+m+\nu_\theta}\right)(a/b)^{3+m-\alpha_1} + (\alpha_1 - \nu_\theta)(a/b)^{\beta} \qquad (2.30)$$

and

$$\beta_3 = (\alpha_1 - \nu_\theta)(a/b)^{1-\alpha_2} - (\alpha_2 - \nu_\theta)(a/b)^{1-\alpha_1} \qquad (2.31)$$

It may be noted that the stress and displacement expressions are not valid for combinations of λ^2, ν_θ, m and β which make

$$(9 - \lambda^2) - \beta(3 + m + \nu_\theta) + m(m+6) = 0 \qquad (2.32)$$

2.8 Assumed Mode Shapes:

Natural frequencies of the disc have been obtained by the Ritz method which is based on the principal of minimum potential and involves determination of maximum potential energy and maximum kinetic energy [25], as well as assumption of appropriate mode shape function, which satisfies atleast the geometric boundary conditions and approximates the actual modes of vibrations.

Assuming that the plate is undergoing harmonic oscillations, the lateral deflection 'w' can be written as

$$w = W_1(r, \theta)\cos \omega_n t \qquad (2.33)$$

and

$W_1(r,\theta)$ is separable as

$$W_1(r, \theta) = W(r) \cos n\theta \qquad (2.34)$$

where n is the number of nodal diameters. Note that the geometric boundary conditions (2.3) and (2.4) depends only upon the function, $W(r)$. The mode shape $W(r)$ in the radial direction is chosen as [6]

$$W(r) = a_o(r-a)^2 + a_1(r-a)^3 + a_2(r-a)^4 + a_3(r-a)^5 + a_4(r-a)^6 + a_5(r-a)^7 \qquad (2.35)$$

where a_o, a_1, a_2, a_3, a_4 and a_5 are unknown constants.

2.9 Maximum Energies:

Substituting for 'w' from Eq.(2.33) in the expressions for potential energies, Eqs.(2.7) and (2.13), we get

$$V_B = \tfrac{1}{2}\cos^2\omega_n t \int_0^{2\pi}\int_a^b [D_r(\frac{\partial^2 W_1}{\partial r^2})^2 + \frac{2D_1}{r}\frac{\partial^2 W_1}{\partial r^2}(\frac{\partial W_1}{\partial r} + \frac{1}{r}\frac{\partial^2 W_1}{\partial \theta^2}) + \frac{D_\theta}{r^2}(\frac{\partial W_1}{\partial r} + \frac{1}{r}\frac{\partial^2 W_1}{\partial \theta^2})^2 + 2D_{r\theta}\{\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial W_1}{\partial \theta})\}^2]rdrd\theta \qquad (2.36)$$

and

$$V_R = \tfrac{1}{2}\cos^2\omega_n t \int_0^{2\pi}\int_a^b \{h\sigma_r(\frac{\partial W_1}{\partial r})^2 + h\sigma_\theta(\frac{\partial W_1}{r\partial \theta})^2\}rdrd\theta \qquad (2.37)$$

The potential energy of the plate is maximum when the displacement is maximum, which occurs when $\cos \omega_n t$ equals unity. This is true when

$$\omega_n t = n_1\pi \qquad (n_1 = 0, 1, 2, 3, \ldots) \tag{2.38}$$

Using these values of $\omega_n t$, the maximum potential energy due to bending and rotation are obtained as

$$V_{B\,max} = \tfrac{1}{2}\int_0^{2\pi}\int_a^b [D_r(\frac{\partial^2 W_1}{\partial r^2})^2 + \frac{2D_1}{r}\frac{\partial^2 W_1}{\partial r^2}(\frac{\partial W_1}{\partial r} + \frac{1}{r}\frac{\partial^2 W_1}{\partial \Theta^2})$$

$$+ \frac{D_\Theta}{r^2}(\frac{\partial W_1}{\partial r} + \frac{1}{r}\frac{\partial^2 W_1}{\partial \Theta^2})^2 + 2D_{r\Theta}\{\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial W_1}{\partial \Theta})\}^2]rdrd\Theta$$

$$\tag{2.39}$$

$$V_{R\,max} = \tfrac{1}{2}\int_0^{2\pi}\int_0^b \{h\sigma_r(\frac{\partial W_1}{\partial r})^2 + h\sigma_\Theta(\frac{1}{r}\frac{\partial W_1}{\partial \Theta})^2\}rdrd\Theta \tag{2.40}$$

Similarly, from Eqs.(2.33) and (2.14) kinetic energy can be written as

$$T = \tfrac{1}{2}\sin^2\omega_n t\ \omega_n^2\int_0^{2\pi}\int_b^b \rho\, h\, W_1^2\, rdrd\Theta \tag{2.41}$$

The kinetic energy is at a maximum, when the velocity of the plate is maximum, which occurs when $w(r,\Theta,t)$ is zero. This will be true if $\cos\omega_n t = 0$; thus

$$\omega_n t = (n_1 + \tfrac{1}{2})\pi \qquad (n_1 = 1,2,3,4,5,\ldots) \tag{2.42}$$

Substituting these values into (2.41), we obtain the expression for maximum kinetic energy as,

$$T_{max} = \frac{\omega_n^2}{2}\int_0^{2\pi}\int_a^b \rho\, h\, W_1^2\, rdrd\Theta \tag{2.43}$$

Substituting Eq.(2.34) in Eqs.(2.39), (2.40) and (2.43), we get

$$V_{B\,max} = \tfrac{1}{2}\int_0^{2\pi}\int_a^b \Big[\cos^2 n\theta\,\Big\{D_r\Big(\frac{\partial^2 W}{\partial r^2}\Big)^2 + \frac{2D_1}{r}\frac{\partial^2 W}{\partial r^2}\Big(\frac{\partial W}{\partial r} - \frac{n^2}{r}W\Big)$$

$$+ \frac{D_\theta}{r^2}\Big(\frac{\partial W}{\partial r} - \frac{n^2}{r}W\Big)^2\Big\} + \sin^2 n\theta\,\frac{4D_{r\theta}\,n^2}{r^2}\Big(\frac{\partial W}{\partial r} - \frac{W}{r}\Big)^2\Big] r\,dr\,d\theta \tag{2.44}$$

$$V_{R\,max} = \tfrac{1}{2}\int_0^{2\pi}\int_a^b \Big\{h\sigma_r\cos^2 n\theta\Big(\frac{\partial W}{\partial r}\Big)^2 + h\sigma_\theta n^2\sin^2 n\theta\,(W/r)^2\Big\} r\,dr\,d\theta \tag{2.45}$$

and

$$T_{max} = \tfrac{1}{2}\,\omega_n^2\int_0^{2\pi}\int_a^b \rho\, h\cos^2 n\theta\, W^2 r\,dr\,d\theta \tag{2.46}$$

Therefore, since

$$\int_0^{2\pi}\sin^2 n\theta\,d\theta = \int_0^{2\pi}\cos^2 n\theta\,d\theta = \pi \tag{2.47}$$

we get

$$V_{B\,max} = \frac{\pi}{2}\int_a^b \Big[D_r\Big(\frac{d^2W}{dr^2}\Big)^2 + \frac{2D_1}{r}\frac{d^2W}{dr^2}\Big(\frac{dW}{dr} - \frac{n^2}{r}W\Big)$$

$$+ \frac{D_\theta}{r^2}\Big(\frac{dW}{dr} - \frac{n^2}{r}W\Big)^2 + \frac{4D_{r\theta}\,n^2}{r^2}\Big(\frac{dW}{dr} - \frac{W}{r}\Big)^2\Big] r\,dr \tag{2.48}$$

$$V_{R\,max} = \frac{\pi}{2}\int_a^b \Big[h\sigma_r\Big(\frac{dW}{dr}\Big)^2 + h\sigma_\theta n^2 (W/r)^2\Big] r\,dr \tag{2.49}$$

and

$$T_{max} = \frac{\pi}{2}\,\omega_n^2\int_a^b \rho\, hW^2 r\,dr \tag{2.50}$$

2.10 Non-dimensionalization

To express energy expressions (2.48 - 2.50) in non-dimensionalised form, we set

$$r = b\bar{r} \tag{2.51}$$

Substituting Eqs.(2.1) and (2.2) for h and ρ, and expressing D_r, D_θ, $D_{r\theta}$ and D_1 in the following form,

$$D_r = \bar{D}_r \bar{r}^{-3\beta} \tag{2.52}$$

$$D_\theta = \bar{D}_\theta \bar{r}^{-3\beta} \tag{2.53}$$

$$D_{r\theta} = \bar{D}_{r\theta} \bar{r}^{-3\beta} \tag{2.54}$$

$$D_1 = \bar{D}_1 \bar{r}^{-3\beta} \tag{2.55}$$

where,

$$\bar{D}_r = \frac{E_r h_o^3}{12(1-\nu_r \nu_\theta)} \tag{2.56}$$

$$\bar{D}_\theta = \frac{E_\theta h_o^3}{12(1-\nu_r \nu_\theta)} \tag{2.57}$$

$$\bar{D}_{r\theta} = \frac{G h_o^3}{12} \tag{2.58}$$

$$\bar{D}_1 = \nu_r \bar{D}_\theta = \nu_\theta \bar{D}_r \tag{2.59}$$

The expressions for maximum energies can now be written as,

$$V_{B\,max} = \frac{\pi}{2}\frac{1}{b^2}\int_{a/b}^{1}\Big[\bar{D}_r\Big(\frac{d^2W}{d\bar{r}^2}\Big)^2 + \frac{2\bar{D}_1}{\bar{r}}\Big(\frac{d^2W}{d\bar{r}^2} - \frac{n^2}{\bar{r}}W\Big)$$

$$+ \frac{\bar{D}_\theta}{\bar{r}^2}\Big(\frac{dW}{d\bar{r}} - \frac{n^2}{\bar{r}}W\Big)^2 + 4\bar{D}_{r\theta}\frac{n^2}{\bar{r}^2}\Big(\frac{dW}{d\bar{r}} - \frac{W}{\bar{r}}\Big)^2\Big]\bar{r}^{1-3\beta}\,d\bar{r} \tag{2.60}$$

$$V_{R\,max} = \frac{\pi}{2}h_o\int_{a/b}^{1}\Big[\sigma_r\Big(\frac{dW}{d\bar{r}}\Big)^2 + n^2\sigma_\theta(W/\bar{r})^2\Big]\bar{r}^{1-\beta}\,d\bar{r} \tag{2.61}$$

and

$$T_{max} = \frac{\pi}{2}b^2\int_{a/b}^{1}\rho_o h_o W^2\,\bar{r}^{1+m-\beta}\,d\bar{r} \tag{2.62}$$

Setting

$$\bar{r} - \bar{a} = x, \tag{2.63}$$

where $\bar{a} = a/b$ (2.64)

Now Eqs.(2.60), (2.61) and (2.62) can be written as

$$V_{B\,max} = \frac{\pi}{2}\frac{\bar{D}_r}{b^2}\int_0^{1-\bar{a}}\Big[\Big(\frac{d^2W}{dx^2}\Big)^2 + \frac{\bar{D}_1}{\bar{D}_r}\frac{2}{(x+\bar{a})}\frac{d^2W}{dx^2}\Big(\frac{dW}{dx} - \frac{n^2}{(x+\bar{a})}W\Big)$$

$$+ \frac{\bar{D}_\theta}{\bar{D}_r}\frac{1}{(x+\bar{a})^2}\Big(\frac{dW}{dx} - \frac{n^2}{(x+\bar{a})}W\Big)^2 + \frac{4\bar{D}_{r\theta}}{\bar{D}_r}\frac{n^2}{(x+\bar{a})^2}$$

$$\Big(\frac{dW}{d\bar{x}} - \frac{W}{(x+\bar{a})}\Big)^2\Big](x+\bar{a})^{1-3\beta}\,dx \tag{2.65}$$

Similarly,

$$V_{R\,max} = \frac{\pi}{2}h_o\int_0^{1-\bar{a}}\Big[(x+\bar{a})^{1-\beta}\sigma_r\Big(\frac{dW}{dx}\Big)^2 + \frac{n^2}{(x+\bar{a})^{1+\beta}}\sigma_\theta W^2\Big]dx \tag{2.66}$$

and

$$T_{max} = \frac{\pi}{2}\,\omega_n^2\, b^2\, \rho_o\, h_o \int_0^{1-\bar{a}} W^2 (x+\bar{a})^{1+m-\beta}\, dx \qquad (2.67)$$

The assumed mode shape (2.35) takes the form

$$W(r) = a_o x^2 + a_1 x^3 + a_2 x^4 + a_3 x^5 + a_4 x^6 + a_5 x^7 \qquad (2.68)$$

In the same manner expressions for radial and tangential stresses can be written as

$$\sigma_r = \rho_o\, \Omega^2 b^2 \Big[C_3 \frac{\beta_1}{\beta_3} \frac{(x+\bar{a})^{\alpha_1+\beta-1}}{(\bar{a})^{\alpha_1+\beta-1}} - C_3 \frac{\beta_2}{\beta_3} \frac{(x+\bar{a})^{\alpha_2+\beta-1}}{(\bar{a})^{\alpha_2+\beta-1}}$$

$$+ C_3 (x+\bar{a})^{2+m}\Big] \qquad (2.69)$$

and

$$\sigma_\theta = \rho_o\, \Omega^2 b^2 \Big[C_3 \frac{\alpha_1\beta_1}{\beta_3} \frac{(x+\bar{a})^{\alpha_1+\beta-1}}{(\bar{a})^{\alpha_1+\beta-1}} - C_3 \frac{\beta_2\alpha_2}{\beta_3} \frac{(x+\bar{a})^{\alpha_2+\beta-1}}{(\bar{a})^{\alpha_2+\beta-1}}$$

$$+ C_3 (3+m+\beta)(x+\bar{a})^{2+m} + (x+\bar{a})^{2+m}\Big] \qquad (2.70)$$

following non-dimensionalized terms are defined,

$$\bar{\sigma}_r = \frac{\sigma_r}{\rho_o\, \Omega^2\, b^2}, \text{ non-dimensional radial stress} \qquad (2.71)$$

$$\bar{\sigma}_\theta = \frac{\sigma_\theta}{\rho_o\, \Omega^2\, b^2}, \text{ non-dimensional tangential stress} \qquad (2.72)$$

$$\bar{\Omega} = \sqrt{\left(\frac{\Omega^2 \rho_0 h_0 b^4}{8 \bar{D}_r} \right)}, \text{ non-dimensional speed} \tag{2.73}$$

$$\bar{\omega}_n = \sqrt{\left(\frac{\omega_n^2 \rho_0 h_0 b^4}{\bar{D}_r} \right)}, \text{ non-dimensional frequency} \tag{2.74}$$

2.11 Determination of Natural Frequencies :

Substituting the expressions for the mode shape from Eqn. (2.68) and for stresses from the Eqns.(2.69) and (2.70) in the expressions for maximum energy Eqns.(2.65-2.67) we obtain

$$V_{Bmax} = \frac{\pi}{2} \frac{1}{b^2} \bar{D}_r f_1 (a_i, E_\theta, E_r, \nu_\theta, \bar{a}, \bar{r}, \beta, m, n) \tag{2.75}$$

$$V_{Rmax} = \frac{\pi}{2} h_0 \rho_0 \Omega^2 b^2 f_2(a_i, E_\theta, E_r, \nu_\theta, \bar{a}, \bar{r}, \beta, m, n) \tag{2.76}$$

and

$$T_{max} = \frac{\pi}{2} h_0 \rho_0 \omega_n^2 b^2 f_3 (a_i, \bar{a}, \bar{r}, \beta, m) \tag{2.77}$$

Applying Ritz-method for obtaining frequencies, we may write

$$\frac{\partial \Pi}{\partial a_i} = 0 \quad (i = 0,1,2 \ldots.5) \tag{2.78}$$

where

$$\Pi = V_{Bmax} + V_{Rmax} - T_{max} \tag{2.79}$$

from Eqns. (2.75-2.77) we obtain,

$$\frac{\partial V_{Bmax}}{\partial a_i} + \frac{h_0 \rho_0 \Omega^2 b^4}{\bar{D}_r} \frac{\partial V_{Rmax}}{\partial a_i} - \frac{h_0 \rho_0 \omega_n^2 b^4}{\bar{D}_r} \frac{\partial T_{max}}{\partial a_i} = 0 \tag{2.80}$$

from Eqn. (2.80) we obtained,

$$[V_B]\{X\} + \bar{\Omega}^2[V_R]\{X\} = \bar{\omega}_n^2 [T_{max}]\{X\} \qquad (2.81)$$

where $[V_B]$, $[V_R]$ and $[T_{max}]$ are 6 x 6 matrices and

$$\{X\} = \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{Bmatrix} \qquad (2.82)$$

Eqn. (2.81) can also be written as

$$[K]\{X\} = \bar{\omega}_n^2 [M]\{X\} \qquad (2.83)$$

where

$$[K] = [V_B] + \bar{\Omega}^2[V_R] \qquad (2.84)$$

and

$$[M] = [T_{max}] \qquad (2.85)$$

Eqn. (2.83) can be premultiplied by $[M]^{-1}$ to give

$$[M]^{-1}[K]\{X\} = \bar{\omega}_n^2 \{X\} \qquad (2.86)$$

and can be reduced into an eigen value equation,

$$[[\Lambda] - \bar{\omega}_n^2 [I]]\{X\} = 0 \qquad (2.87)$$

Natural frequencies are obtained from the roots of eqn. (2.87).

CHAPTER 3

RESULTS AND DISCUSSIONS

Natural frequencies of orthotropic discs have been calculated numerically from Eqn.(2.87). The effect of variation of clamping radius $\frac{a}{b}$, thickness parameter β, density parameter m, rotational speed $\bar{\Omega}$, rigidity parameter λ^2 and poisson's ratio have been studied. Frequencies have been obtained for 0 to 5 nodal circles and for 0 to 4 nodal diameters. Frequencies plotted in Figs.2, 7-8, 10-14 and in 16-18 are for zero nodal circle and zero nodal diameter mode. The parameter $\bar{D}_{r\theta}/\bar{D}_r$ was fixed at 0.35 and Poisson's ratio ν_θ was fixed at 0.3 (except in the case, in which the effect of variation of ν_θ is studied). Computations were carried out with double precision arithematic for all type of discs. Comparison have been made with available results for isotropic and orthotropic discs. The frequencies obtained in the present analysis is in good agreement with previous work.

3.1 Effect of Variation of Clamping Radius

Natural frequencies of a stationary, isotropic (λ^2=1.0) disc of constant thickness and constant density have been obtained, for various clamping radius $\frac{a}{b}$ and compared with Mote [6], (Table -1). It is found that present frequency estimation compares well with that of Mote. Frequencies obtained

for above case have also been compared with exact natural frequencies given by Southwell [2]. Approximate and exact natural frequencies for various clamping geometries and at different nodal circles and diameters are shown in Table 2. For $\frac{a}{b} < 0.7$ the present approximate values of the natural frequency compares well with the exact values of southwell (Fig. 2). For $\frac{a}{b} > 0.7$ the error is large, however, this range is not of practical interest. Fig. 2 shows plot of clamping radius vs. natural frequency at zero nodal circle, zero nodal diameter for present, Mote's and Southwell's case.

The variation of natural frequencies with clamping radius $\frac{a}{b}$ is shown in(Fig. 3) in the case of a disc of constant thickness and density, rotating at a speed of $\bar{\Omega} = 20$. The value of the rigidity parameter λ^2 is taken to be 0.25. Figs. 4 and 5 are plotted for the case when λ^2=1.0 and 4.0. Frequencies are plotted for various nodal circles and nodal diameters. From the above analysis it is found that the natural frequency increases as the clamping radius increases from .1 to .8.

3.2 Effect of Variation of Thickness Parameter

Natural frequencies of a stationary isotropic disc (λ^2=1.0)have been obtained for β=0.9 and are compared with the results of the Mote [6] (Table 3.). The comparison is very good. From table 1 and 3 it can be seen that frequency is high for β=0.9 than for β=0.0 (constant thickness). Turbine discs can be

dealised as a disc whose thickness decreases hyperbolically n radial direction (Fig. 1-b) . It is found that as thickness arameter β increases from 0 (constant thickness disc) to a ositive value (that is the disc is becoming thicker at the ub), frequency also increases as can be seen from Figs. 7,10,11 nd 15. On the other hand if β increases negatively (that is the isc is becoming thicker at the periphery) the frequency ecreases. Fig. 16 shows variation of frequency with the nodal iameters for different values of β.

.3 Effect of the Variation of Density Parameter

Use of composites may cause radial variation of density. eeping this in mind natural frequencies of disc having $^2 =$ 1 and 4.00 respectively with different density rameter m, have been determined and plotted in Fig. 8. As e value of parameter m increases (that is density is more wards the periphery) frequencies decrease for $\lambda^2=$ 00 but for $\lambda^2= 4.00$ it slightly increase.

4 Effect of Rotational Speed:

Effect of rotational speed on the natural frequencies isotropic and orthotropic discs have been studied for various sc geometries and density parameter m. Frequencies of an otropic rotating disc of constant thickness and density have en compared with the work of Barasch and Chen [13] and ithwell [2]. Frequencies are plotted for 0, 1 and 2 nodal

circles and for zero nodal diameter (Fig.9), which agree well with the frequencies obtained by -Barasch and Chen and is slightly greater than the Southwell's values. Figures 10 to 14 show the variation of frequency with the rotational speed with λ^2 as a parameters for various values of β and m. As the rotational speed increases frequency also increases. This is because of in-plane stresses due to rotation. Increase in speed increases in-plane stresses which inturn increases stiffness and hence the frequency. Fig.16 shows the variation of frequency of a membrane with rotation.

3.5 Effect of Variation of Rigidity Parameter:

Freqaencies of stationary orthotropic disc of constant thickness and constant density for different clamping geometries have been obtained and compared with the frequencies obtained from the approximate formula given by Ramaiah and Vijay Kumar [22] in Table 4. Freqaencies obtained from the present analysis are in good agreement with those obtained from approximate formula.

Figs. 10 to 14 shows variation of frequencies in the case of a rotating disc for different values of λ^2. It is seen that at lower speed the frequency increases as λ^2 increases but at higher speeds the frequency decreases with increasing λ^2. When a rigid rotating disc is executing bending vibrations, it is controlled both by bending stiffness and tension due to in-plane stresses. Since the tensions in the disc are proportional to the square of the angular velocity, it is clear that at low

speed the main controlling factor is the bending rigidity. Whereas at high speeds the controlling factor is mainly the in-plane stresses. The effect of increasing λ^2 is to increase the natural frequency. The stresses due to rotation decreases with increase in the rigidity parameter λ^2 (which is shown in Fig. 6) and consequently the natural frequency decreases with increase in λ^2. This explains why the natural frequency increases with increasing λ^2 at lower speeds and decreases with increasing λ^2 at higher speeds. In the case of membrane in which the bending rigidity is zero, the variation of natural frequency with speed at different λ^2 is shown in Fig. (17).

3.6 Effect of Poisson's ratio ν_θ :

Fig. 18 shows the variation of frequency with Poisson's ratio ν_θ for a constant thickness, constant density isotropic (λ^2=1) and orthotropic (λ^2=4) discs rotating at speed $\bar{\Omega}$=15.

CONCLUSION

In this investigation the effect of clamping radius, thickness parameter, density parameter, rotational speed, rigidity parameter and Poisson's ratio on the natural frequencies of an orthotropic rotating disc has been studied. The following conclusions are arrived at:

1. As the clamping radius increases frequency increases.
2. As the disc becomes thicker at the hub, frequency increases.
3. As the disc becomes heavier towards periphery the frequency increases or decreases depending upon the rigidity parameter and disc configuration.
4. Increase in the rotational speed leads to the increase in frequency.
5. If the ratio of Young's modulus in tangential direction to the Young's modulus in radial direction increases, frequency increases.
6. Increases in Poisson's ratio lead to an increase or decrease in frequency depending on the value of the rigidity parameter.

Table 1

Flexural natural frequencies, ~~for the three mode shapes~~ comparison with C.D. Mote, Jr.

Radius a/b	Nodal diameter n	Frequency, $\omega_n\sqrt{(\rho b^4/EH^2)}$	
		Present work	C.D.Mote's work
0.1	0	2.5659	2.5706
0.2	0	3.1359	3.7373
0.3	0	4.0310	4.0314
0.4	0	5.4595	5.4597
0.5	0	7.8826	7.8828
0.6	0	12.4203	12.4200
0.7	0	22.3646	22.3620
0.8	0	51.1413	50.9940
0.1	1	2.1099	2.1211
0.2	1	2.9135	2.9164
0.3	1	3.9657	3.9676
0.4	1	5.5169	5.5181
0.5	1	8.0433	8.0447
0.6	1	12.6710	12.6720
0.7	1	22.6949	22.6950
0.8	1	51.5420	51.3980

Continued...

Table 1 continued.

Radius a/b	Nodal diameter n	Frequency, $\omega_n \sqrt{(\rho b^4/EH^2)}$	
		Present work	C.D. Mote's work
0.1	2	3.4077	3.4147
0.2	2	3.9036	3.9196
0.3	2	4.8158	4.8367
0.4	2	6.3335	6.3452
0.5	2	8.8991	8.9051
0.6	2	13.6184	13.6220
0.7	2	23.7714	23.7750
0.8	2	52.7713	52.6400
0.1	3	7.5361	7.5338
0.2	3	7.6360	7.6509
0.3	3	8.0355	8.0644
0.4	3	9.0562	9.0806
0.5	3	11.2342	11.2500
0.6	3	15.7238	15.7330
0.7	3	25.8156	25.8250
0.8	3	54.9052	54.7940

Note: $H = \frac{1}{2}$ peripheral disc thickness.

Table 2

Comparison of approximate flexural natural frequencies with exact flexural natural frequencies - Southwell.

Radius a/b	Nodal diameter n	Frequency, $\omega_n \sqrt{(\rho b^4/EH^2)}$	
		Present work	Southwell's work
0.276	0	3.778	3.78
0.642	0	15.581	15.10
0.840	0	80.49	49.10
0.060	1	1.808	1.71
0.397	1	5.459	5.45
0.603	1	12.866	12.80
0.634	1	15.172	15.10
0.771	1	39.195	38.80
0.827	1	69.100	73.20
0.186	2	3.812	3.78
0.394	2	6.219	5.45
0.522	2	9.690	9.70
0.769	2	39.692	38.80
0.810	2	58.419	60.70
0.430	3	9.553	9.71
0.590	3	15.108	15.15
0.710	3	27.444	29.70
0.810	3	60.568	60.60

Note: $H = \frac{1}{2}$ Peripheral disc thickness.

Table 3

Flexural natural frequencies - Variable thickness disc ($h(r) = H(r/b)^{-0.9}$), Comparison with C.D. Mote, Jr.

Radius a/b	Nodal diameter n	Frequency $\omega_n \sqrt{(\rho b^4/EH^2)}$ Present work	C.D.Mote's work
0.1	0	9.2363	9.2499
0.2	0	9.5081	9.5135
0.3	0	10.2617	10.2650
0.4	0	11.7753	11.7770
0.5	0	14.5742	14.5760
0.6	0	19.9915	19.1330
0.7	0	31.4183	31.4160
0.8	0	63.5736	63.3700
0.1	1	9.0925	9.1027
0.2	1	9.4598	9.4631
0.3	1	10.3140	10.3160
0.4	1	11.9149	11.9170
0.5	1	14.7880	14.7900
0.6	1	20.1912	20.1940
0.7	1	31.7594	31.7590
0.8	1	63.9740	63.7650

Continued...

Table 3 continued.

Radius a/b	Nodal diameter n	Frequency, $\omega_n \sqrt{(\rho b^4/EH^2)}$	
		Present work	C.D.Mote's work
0.1	2	9.7168	9.7230
0.2	2	10.1608	10.1640
0.3	2	11.0862	11.0890
0.4	2	12.7527	12.7550
0.5	2	15.6957	15.6990
0.6	2	21.1829	21.1790
0.7	2	32.8564	32.8580
0.8	2	65.2010	64.9690
0.1	3	13.0749	13.1000
0.2	3	13.2705	13.2750
0.3	3	13.8798	13.8860
0.4	3	15.2408	15.2470
0.5	3	17.9402	17.9490
0.6	3	23.2756	23.2890
0.7	3	34.9070	34.9120
0.8	3	67.3277	67.0250

Notes: (i) $H = \frac{1}{2}$ Peripheral disc thickness.

(ii) All frequencies are for the zero nodal circle mode of vibration.

Table 4

Frequencies of Orthotropic stationary discs of constant thickness and constant density.

Comparison with Ramaiah and Vijay Kumar.

(Zero nodal circle and 1 nodal diameter).

	Clamping radius a/b	Value of constant M	Frequency, $(h_o \rho_o \omega_n^2 b^4)/D_r$	
			Present work	From approx. formula (Ramaiah and Vijay Kumar)
.25	0.1	0.548	1.916	1.931
.25	0.3	0.312	3.530	3.528
.25	0.5	0.140	7.461	7.457
.25	0.7	0.42	22.000	22.009
.33	0.1	0.578	1.998	2.011
.33	0.3	0.312	3.589	3.587
.33	0.5	0.140	7.509	7.505
.33	0.7	0.042	22.041	22.049
.50	0.1	0.548	2.154	2.163
.50	0.3	0.312	3.705	3.703
.50	0.5	0.140	7.604	7.601
.50	0.7	0.042	22.122	22.128
.00	0.1	0.578	2.565	2.565
.00	0.3	0.312	4.031	4.031
.00	0.5	0.140	7.882	7.882
.00	0.7	0.042	22.364	22.364

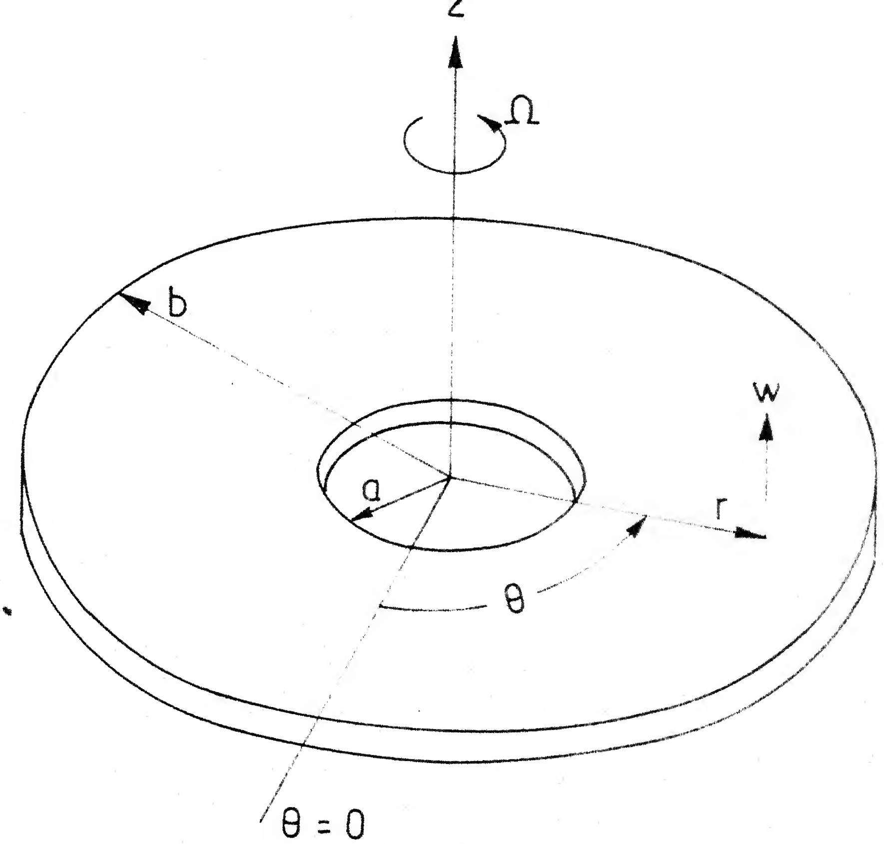


Fig. 1a Schematic illustration of a disc

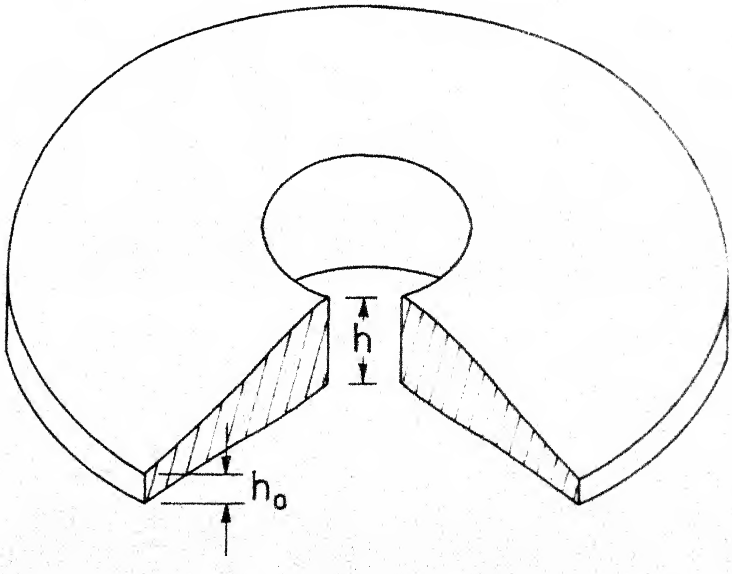


Fig. 1b Disc of variable thickness: $h = h_o(r/b)^{-\beta}$.

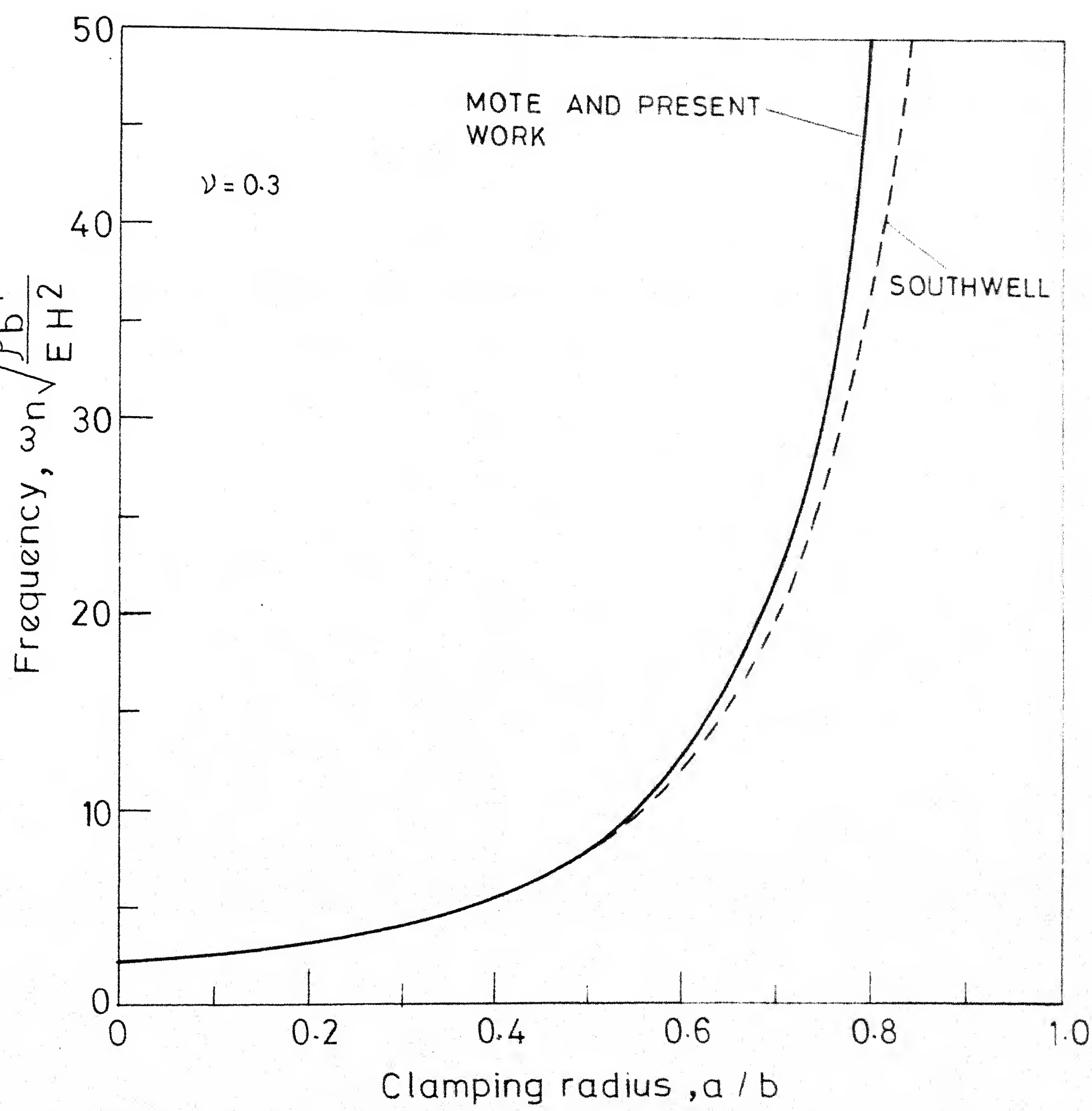


FIG. 2 VARIATION OF FREQUENCY WITH CLAMPING RADIUS (COMPARISON WITH SOUTHWELL)

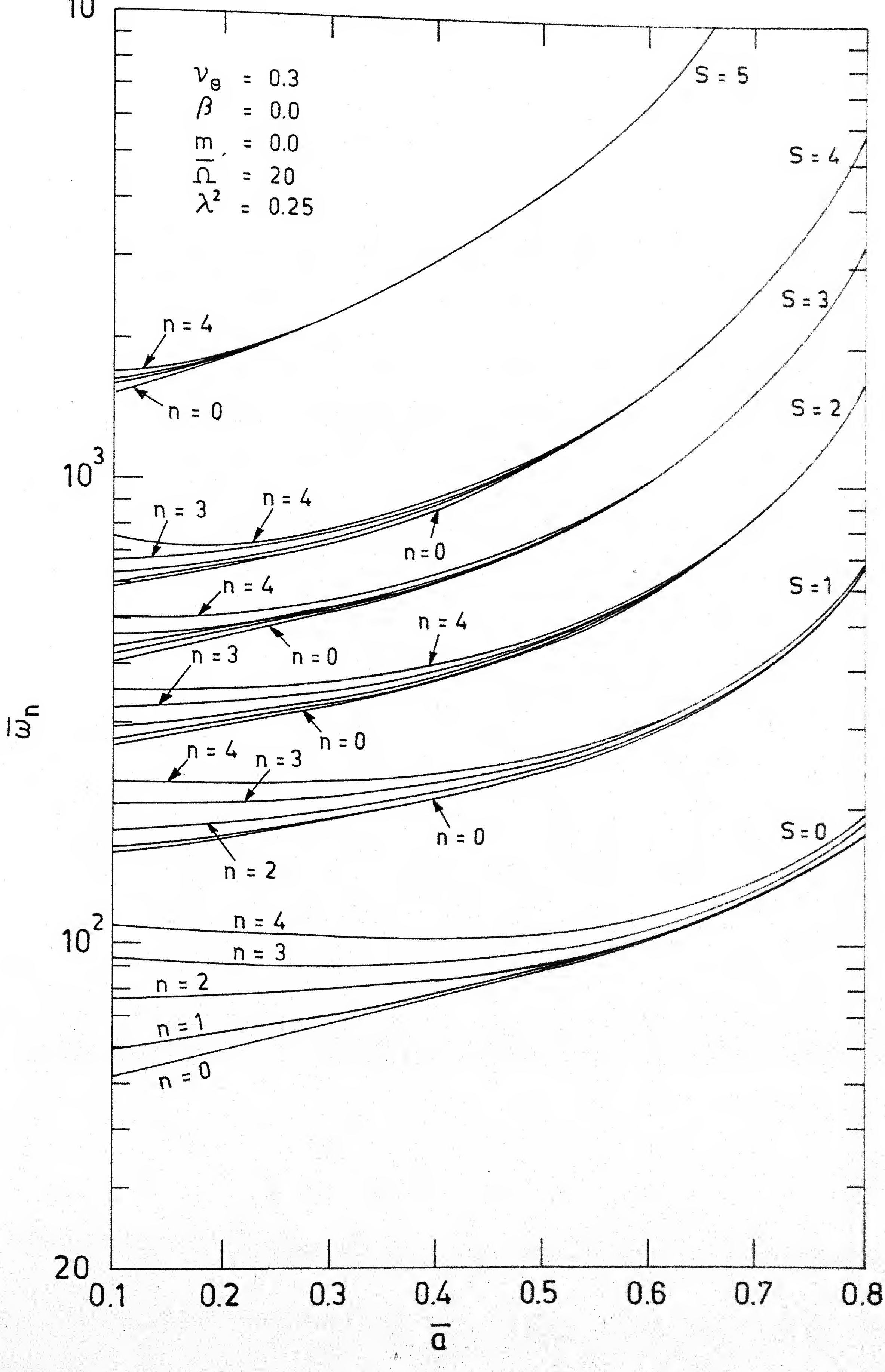


Fig. 3 Variation of frequency with clamping radius.

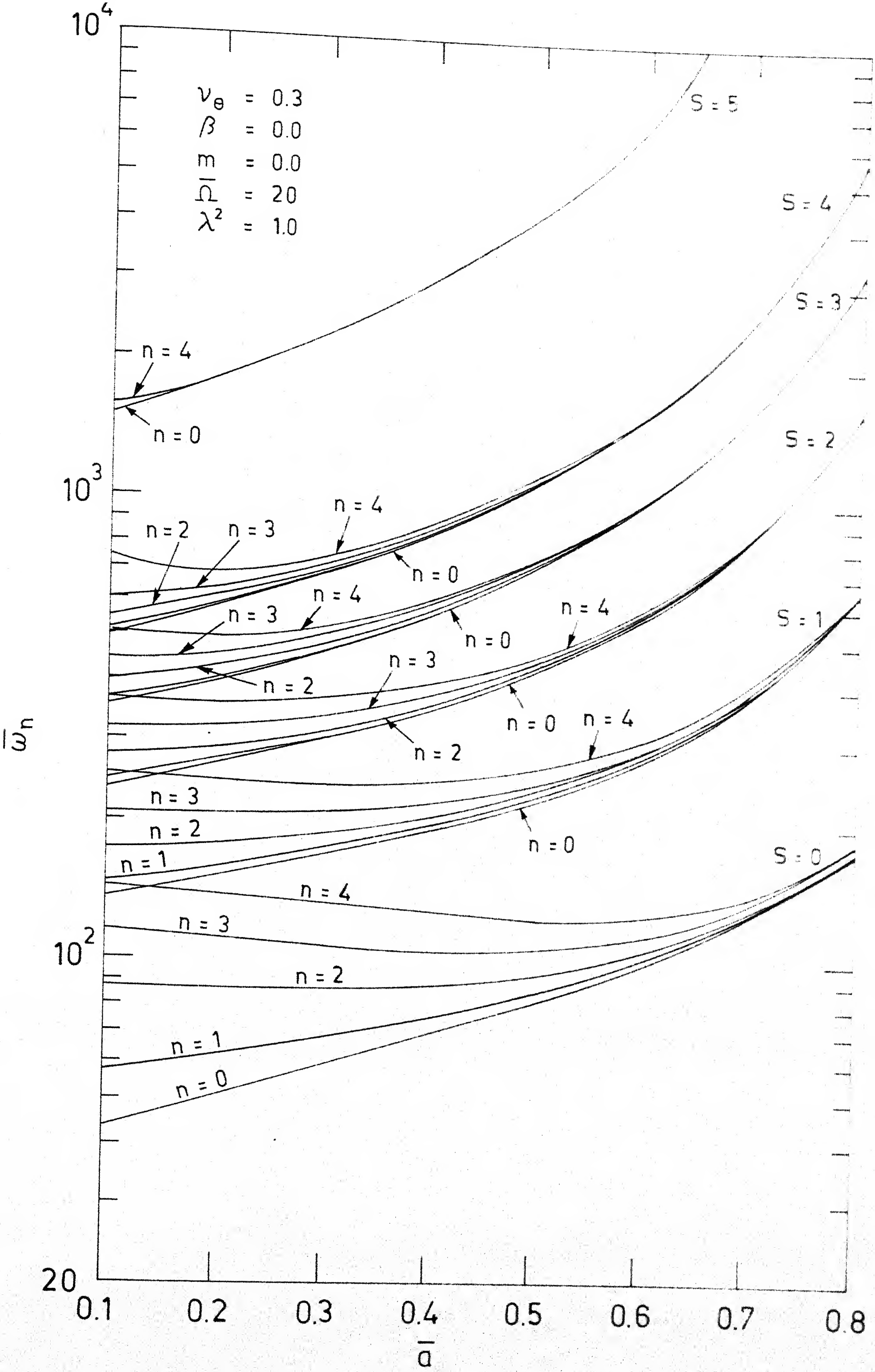
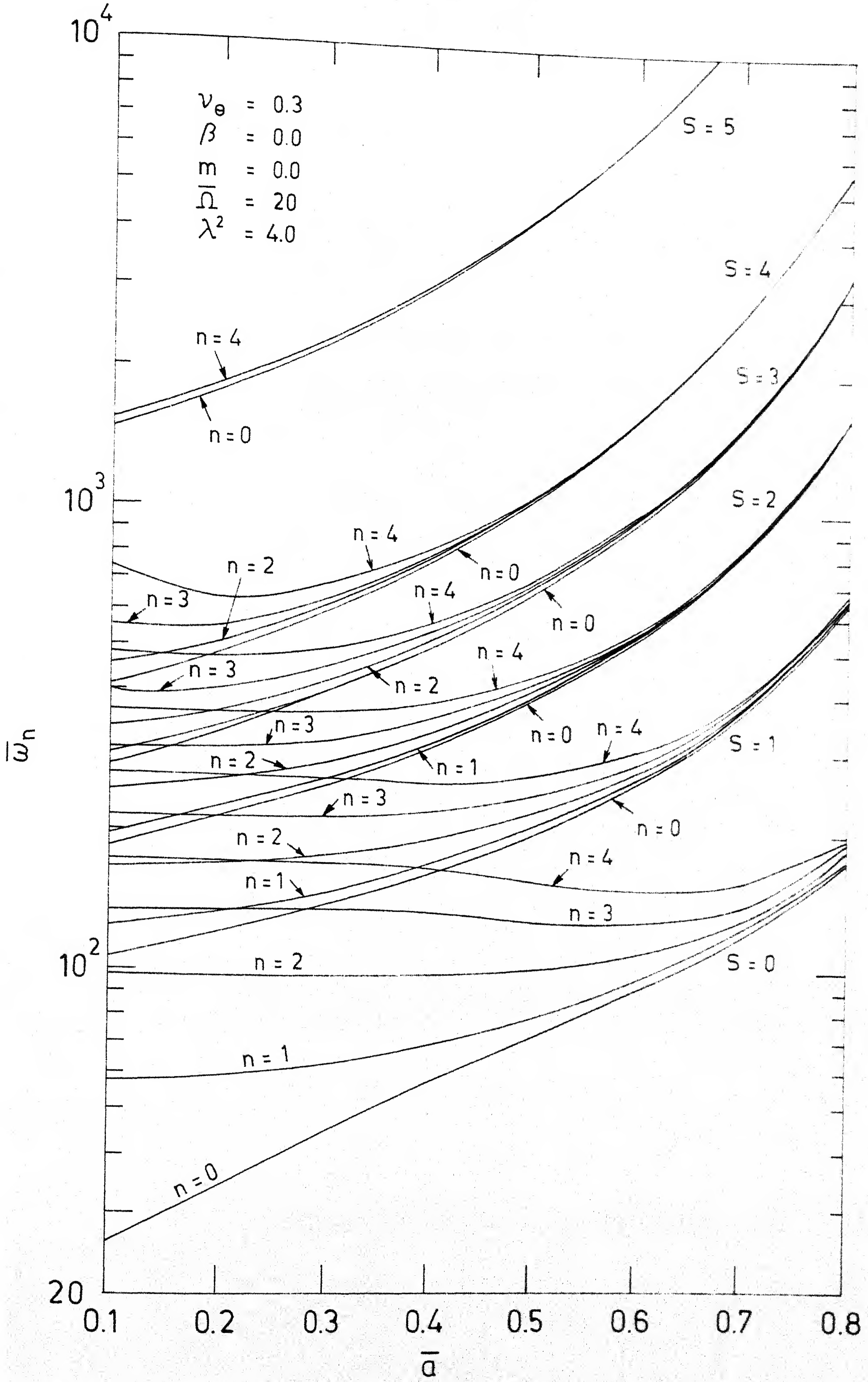
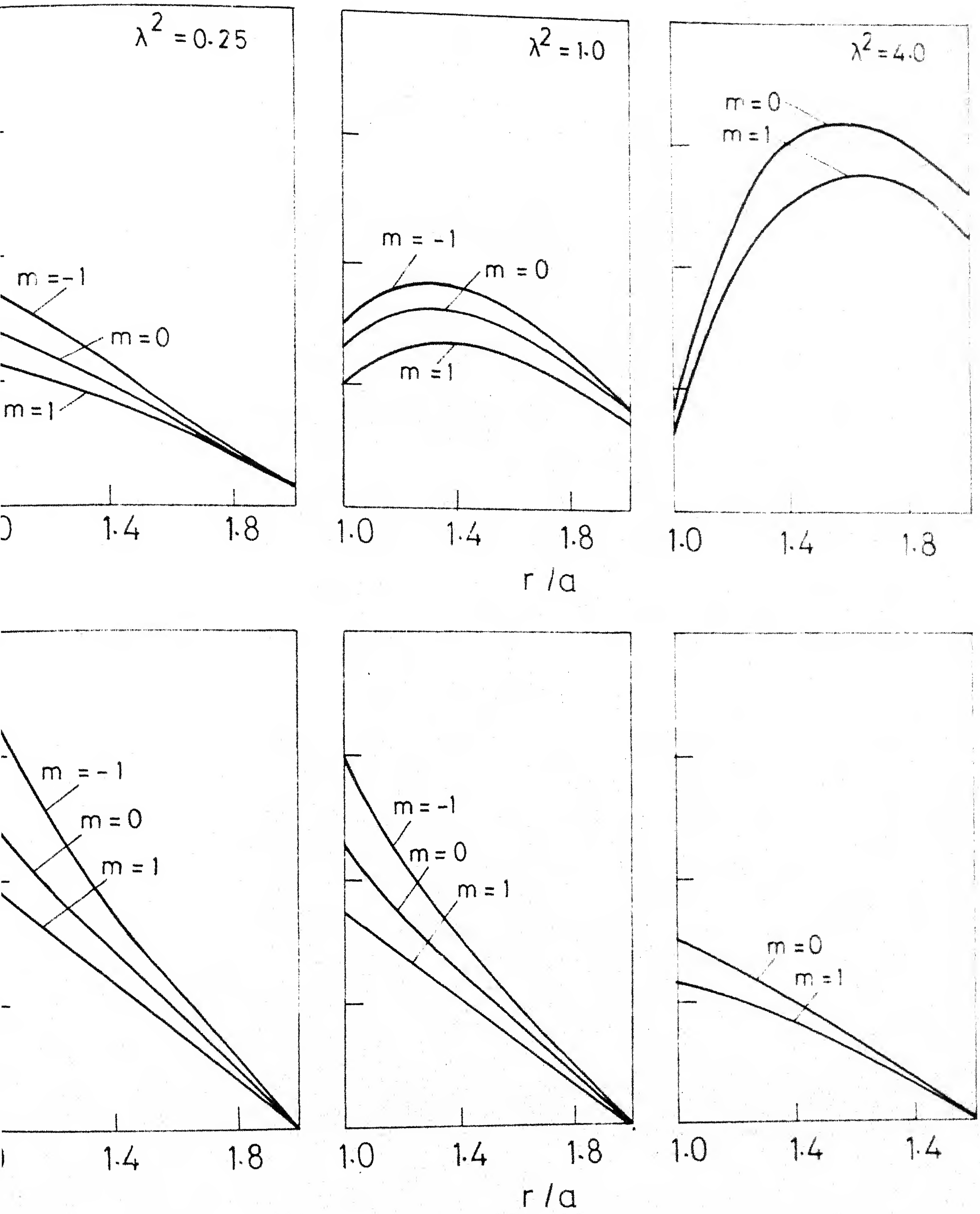


Fig. 4 Variation of frequency with clamping radius.



ν_θ = 0.3
β = 0.0
m = 0.0
$\bar{\Omega}$ = 20
λ^2 = 4.0
S = 5
S = 4
S = 3
S = 2
S = 1
S = 0
n = 0
n = 1
n = 2
n = 3
n = 4
$\bar{\omega}_n$
$\bar{a}$
10^4
10^3
10^2
20
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



STRESS DISTRIBUTION WITH RADIUS ($\nu_\theta = 0.3$, $a/b = 0.5$, $\beta = 0.0$).

~~of frequency with thickness~~

parameter.

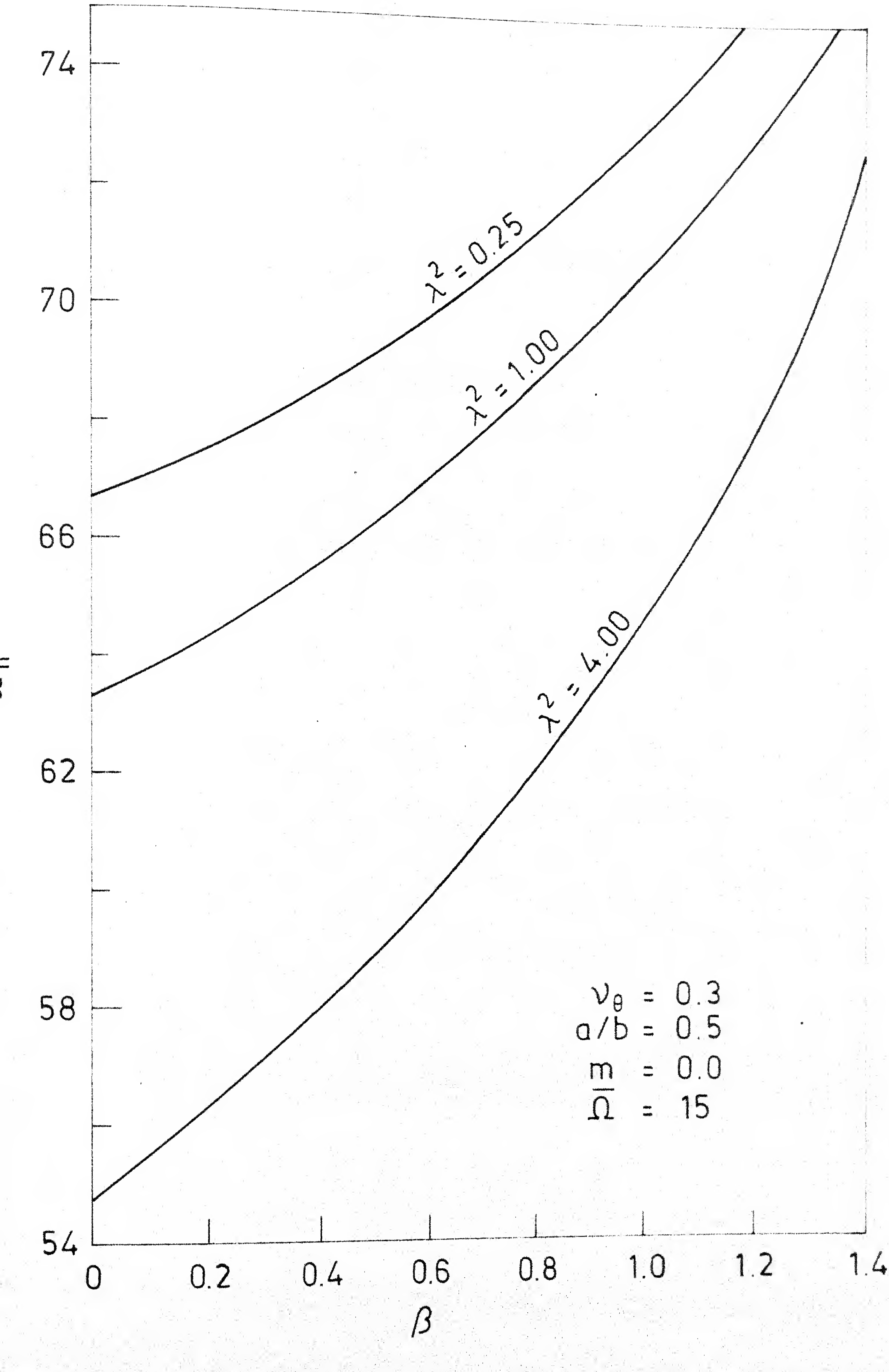


Fig. 7 Variation of frequency with thickness parameter.

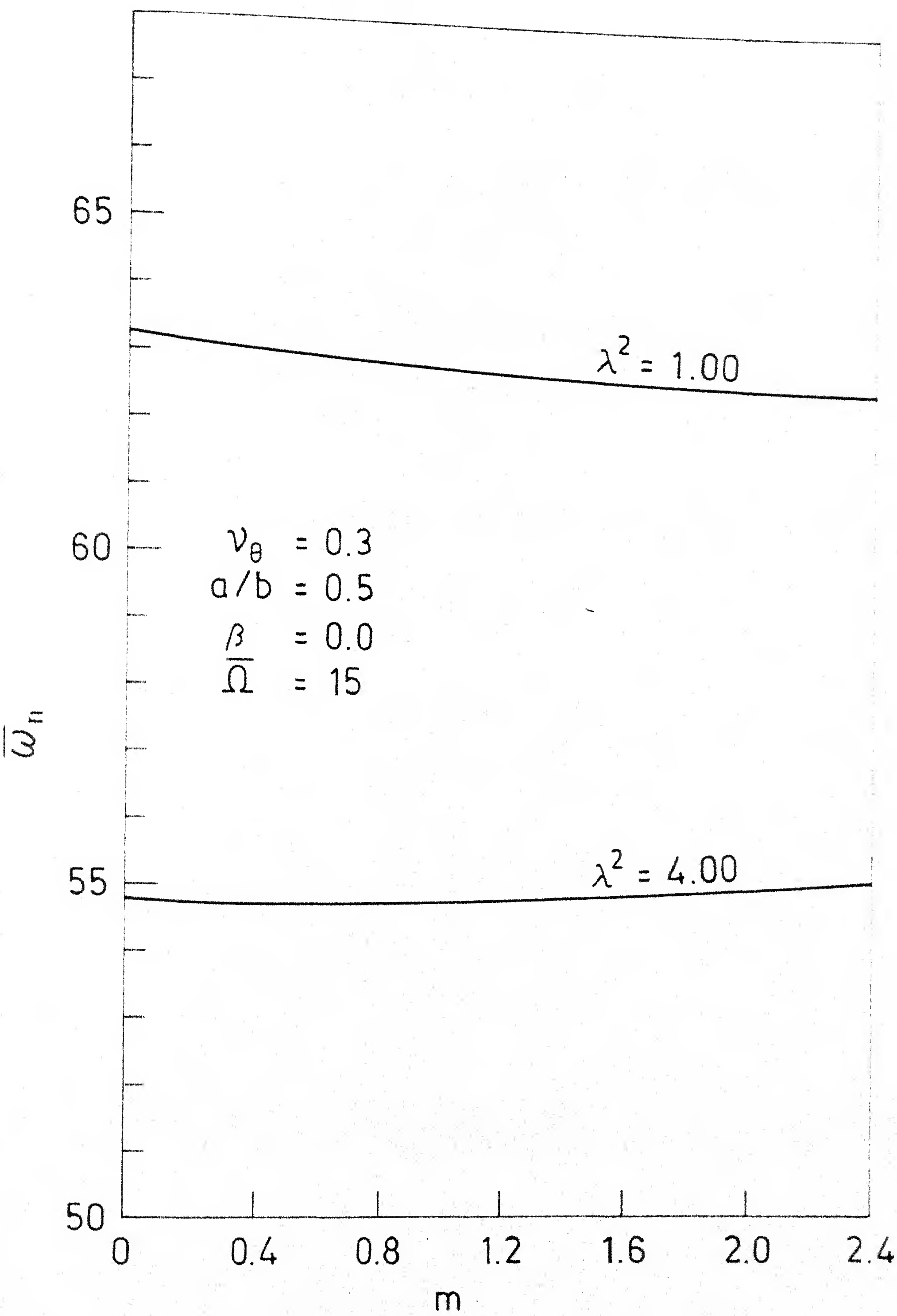
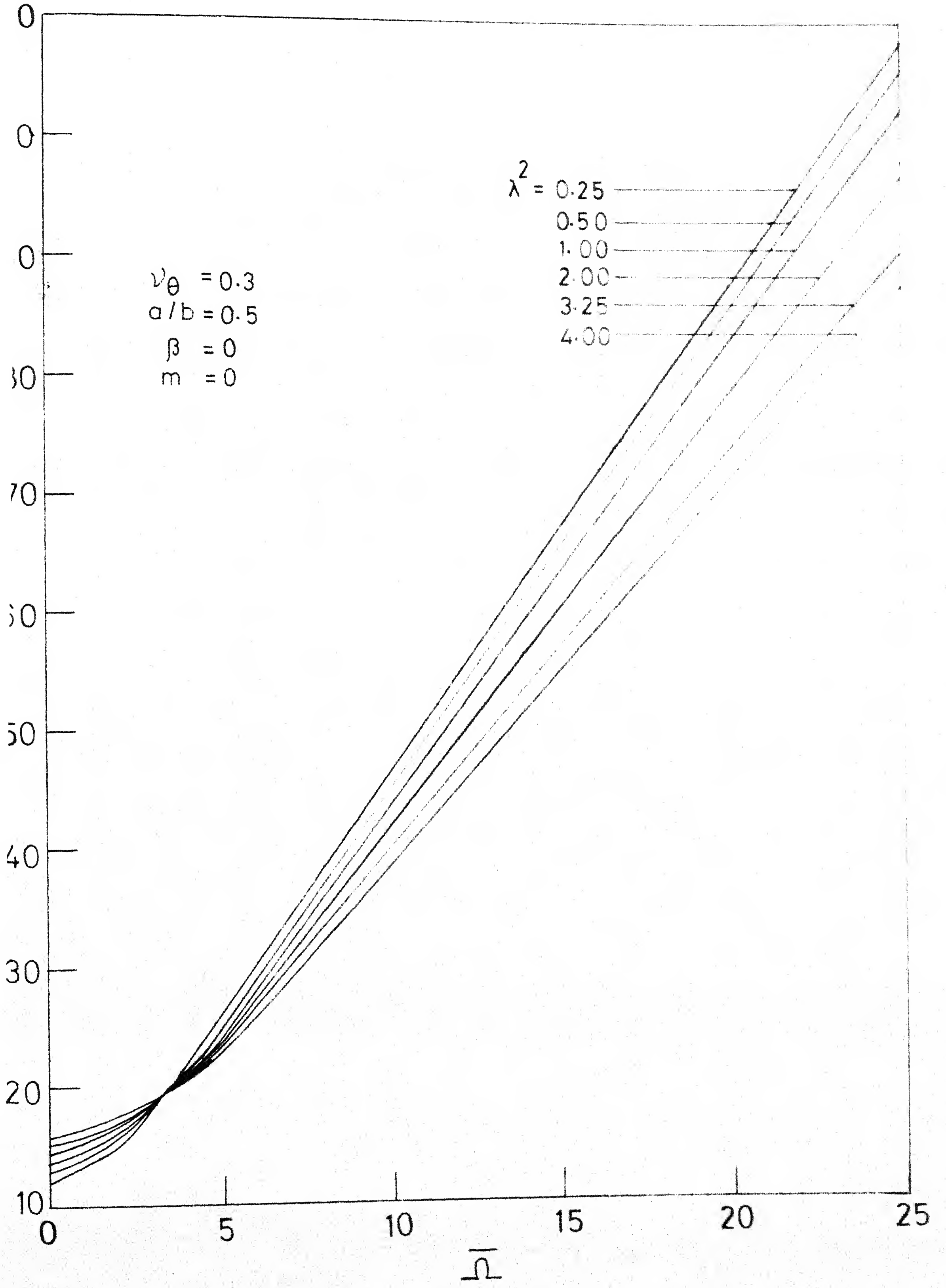
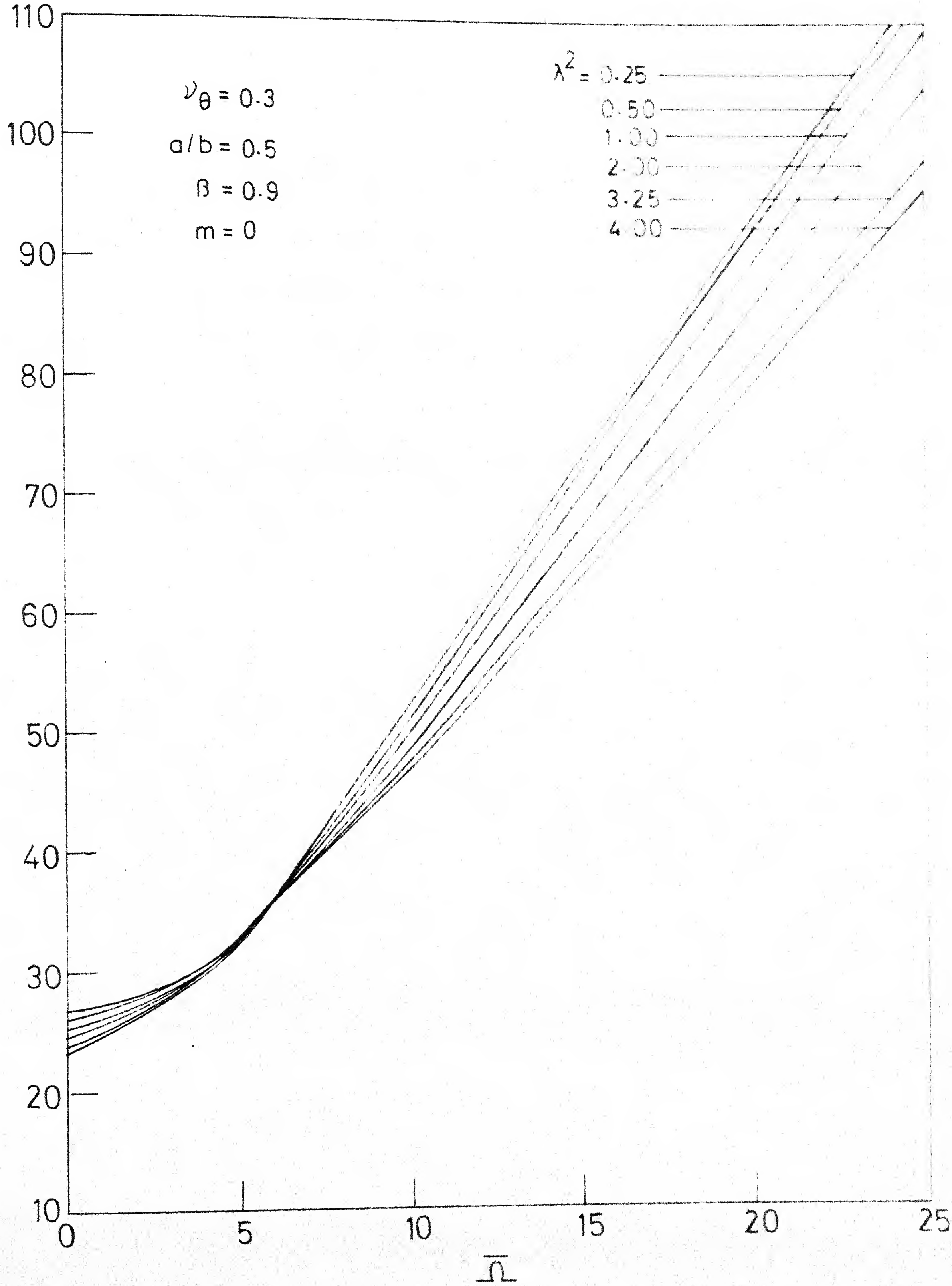


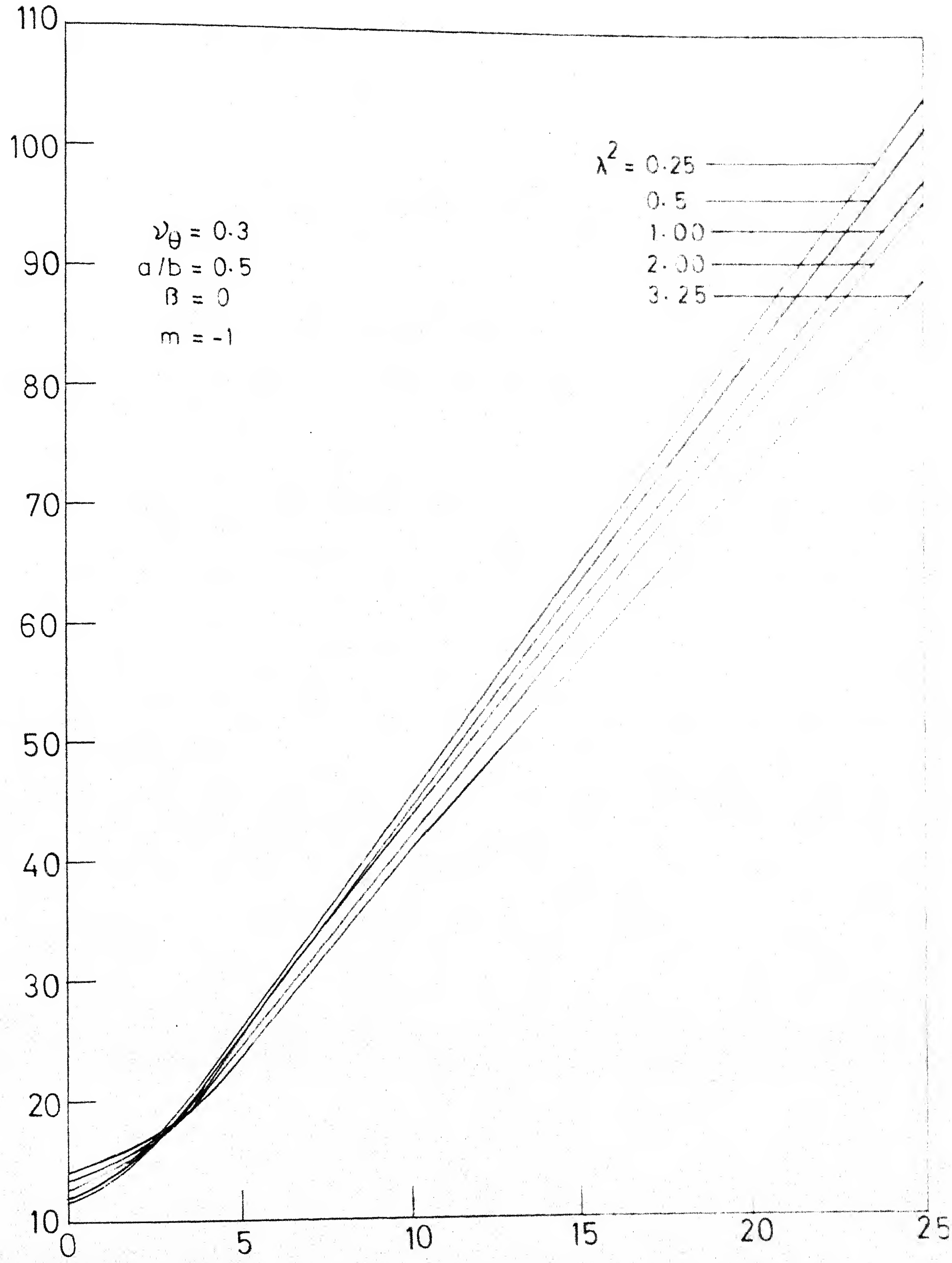
Fig. 8 Variation of frequency with density parameter.



10 VARIATION OF FREQUENCY WITH ROTATION SPEED



G. 11 VARIATION OF FREQUENCY WITH ROTATION SPEED



G. 12 VARIATION OF FREQUENCY WITH ROTATION SPEED

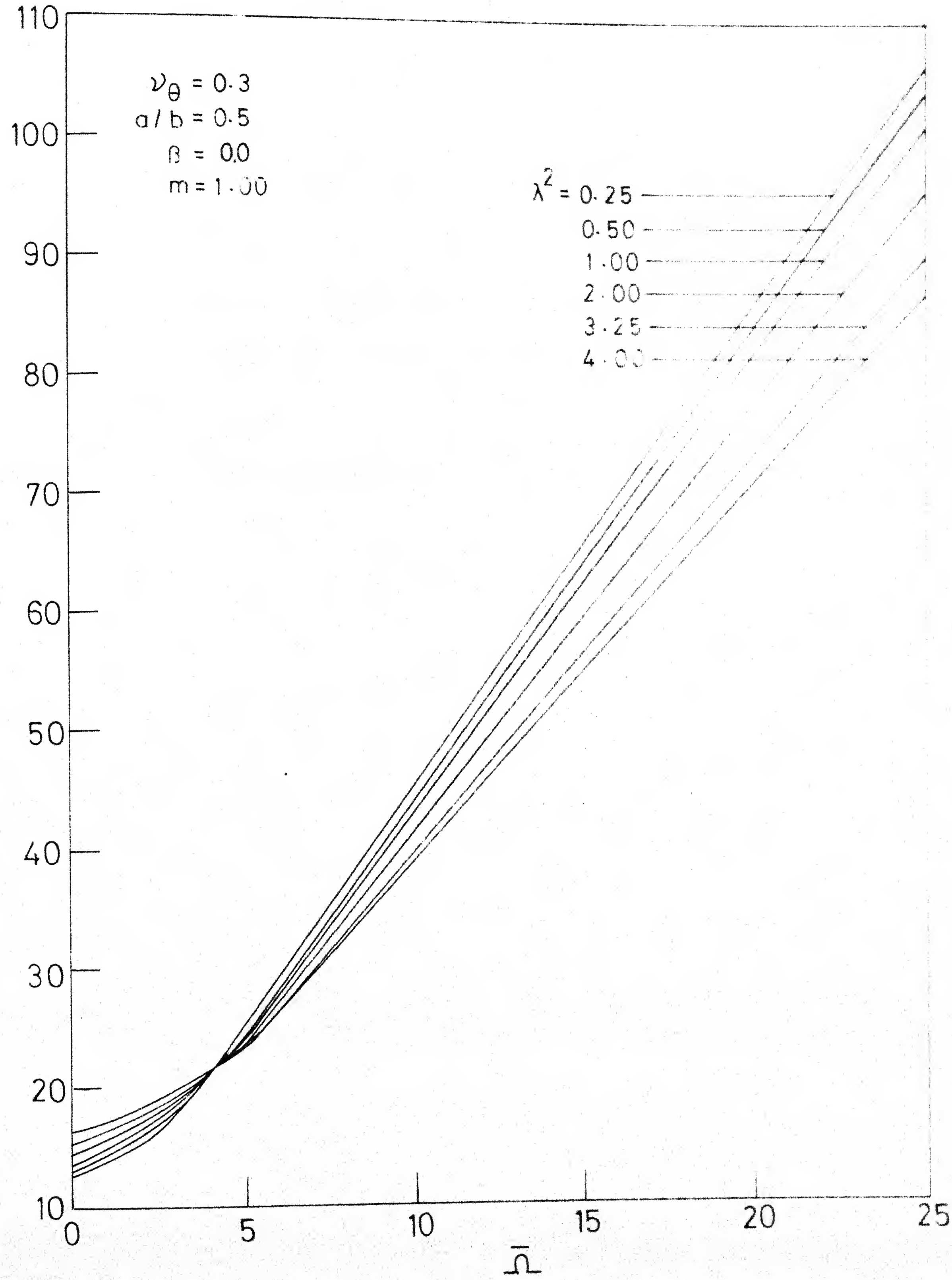


FIG. 13 VARIATION OF FREQUENCY WITH ROTATION SPEED.

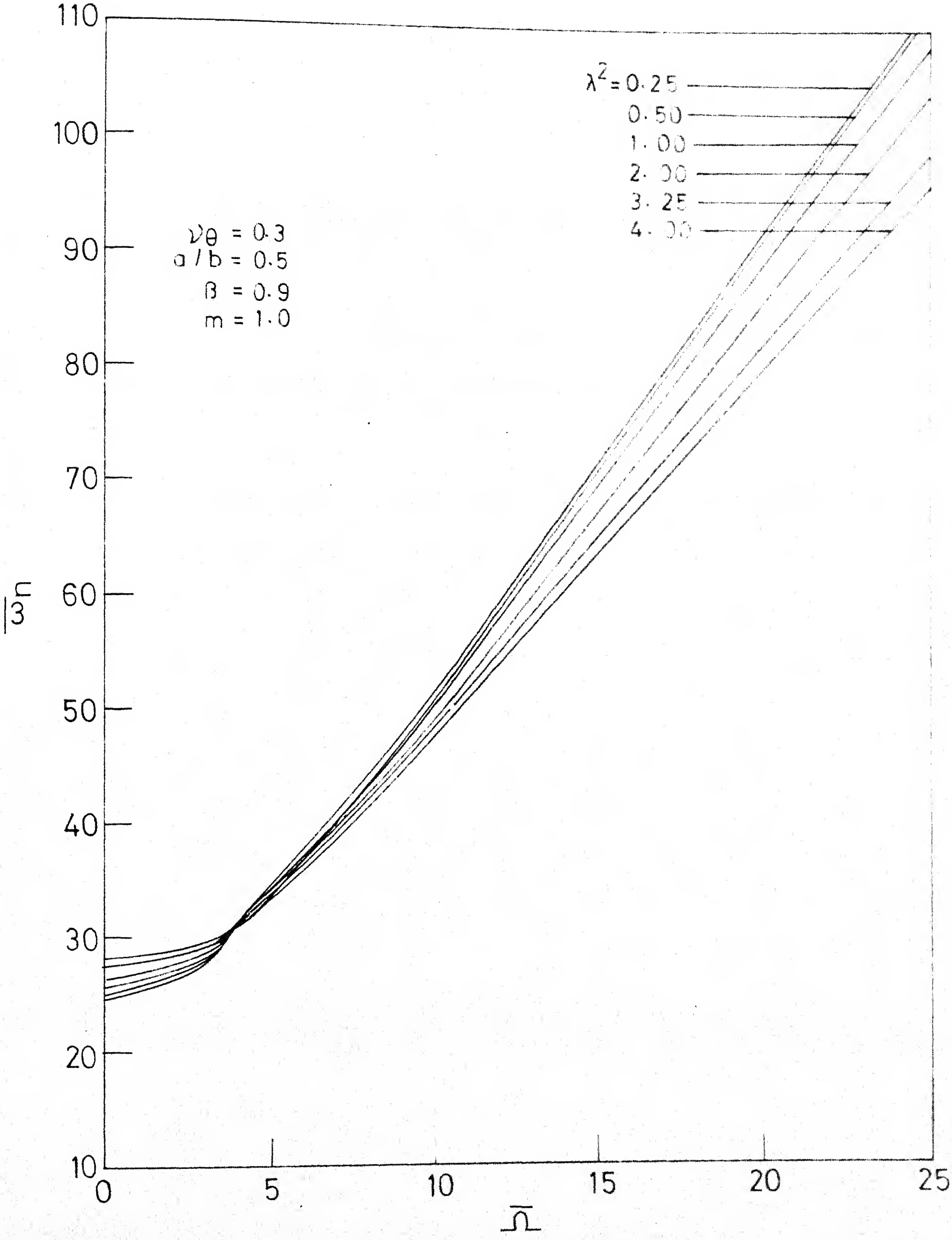


FIG. 14 VARIATION OF FREQUENCY WITH ROTATION SPEED.

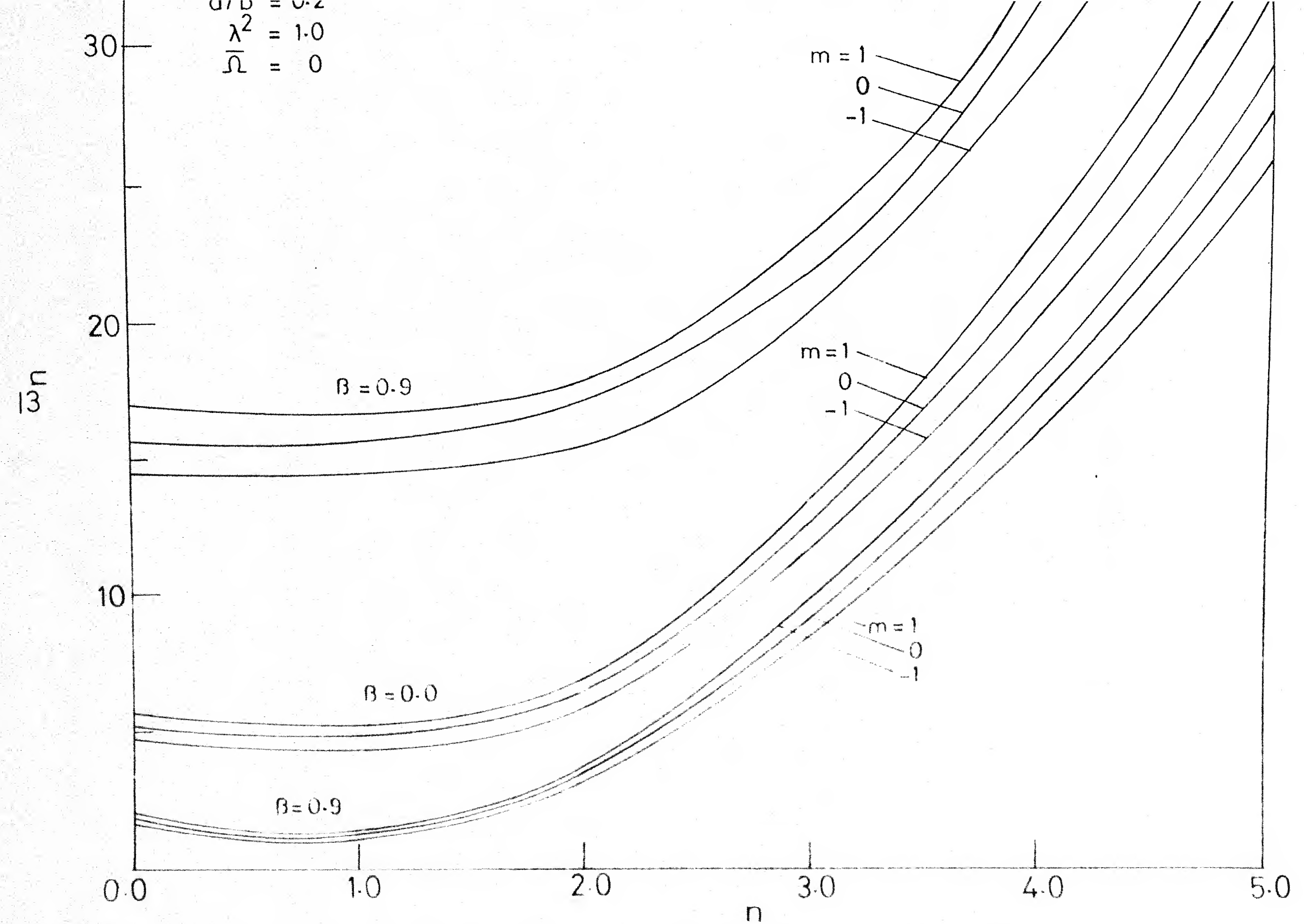


FIG. 15 VARIATION OF FREQUENCY WITH NODAL DIAMETERS

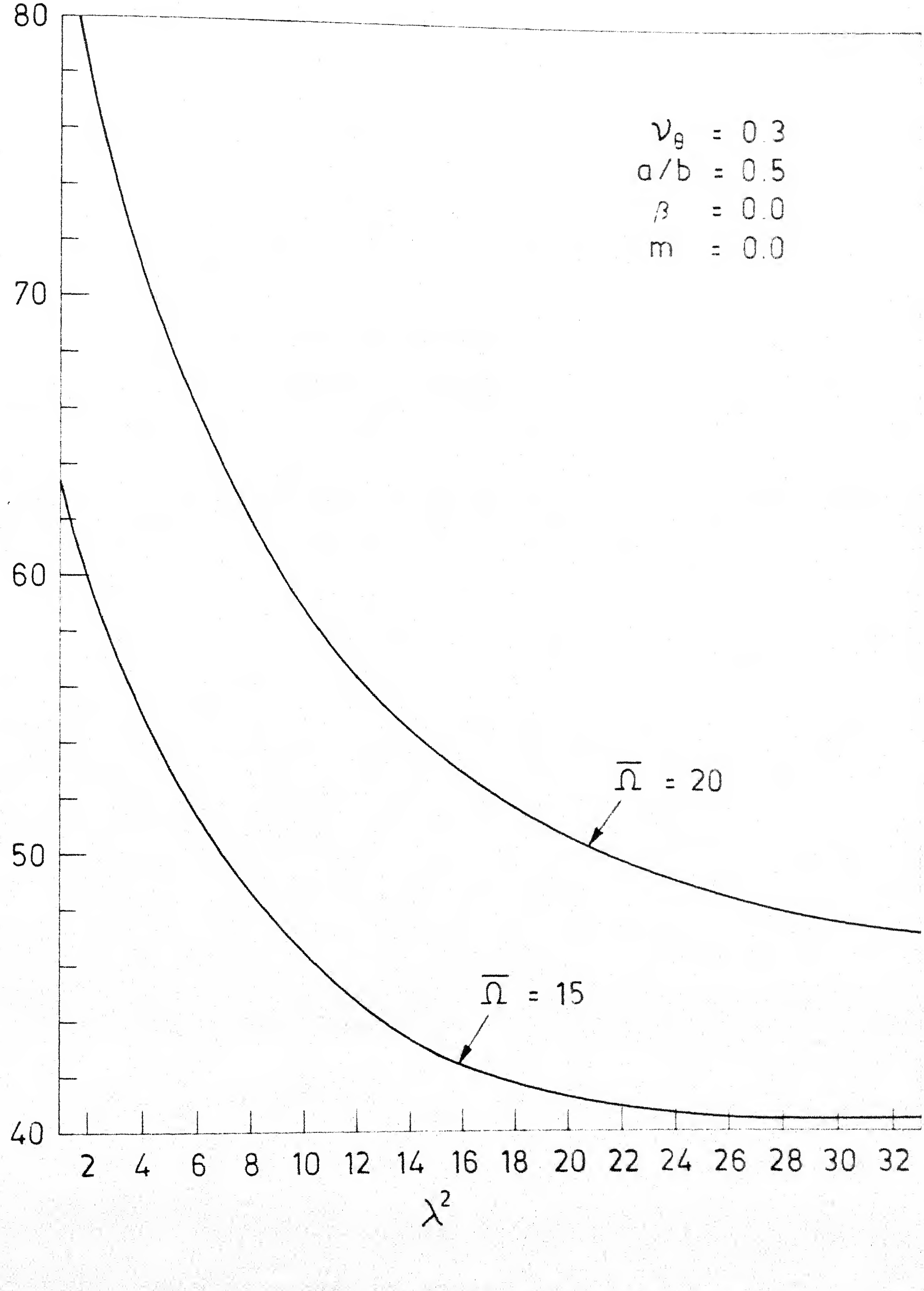


Fig. 16 Variation of frequency with rigidity parameter.

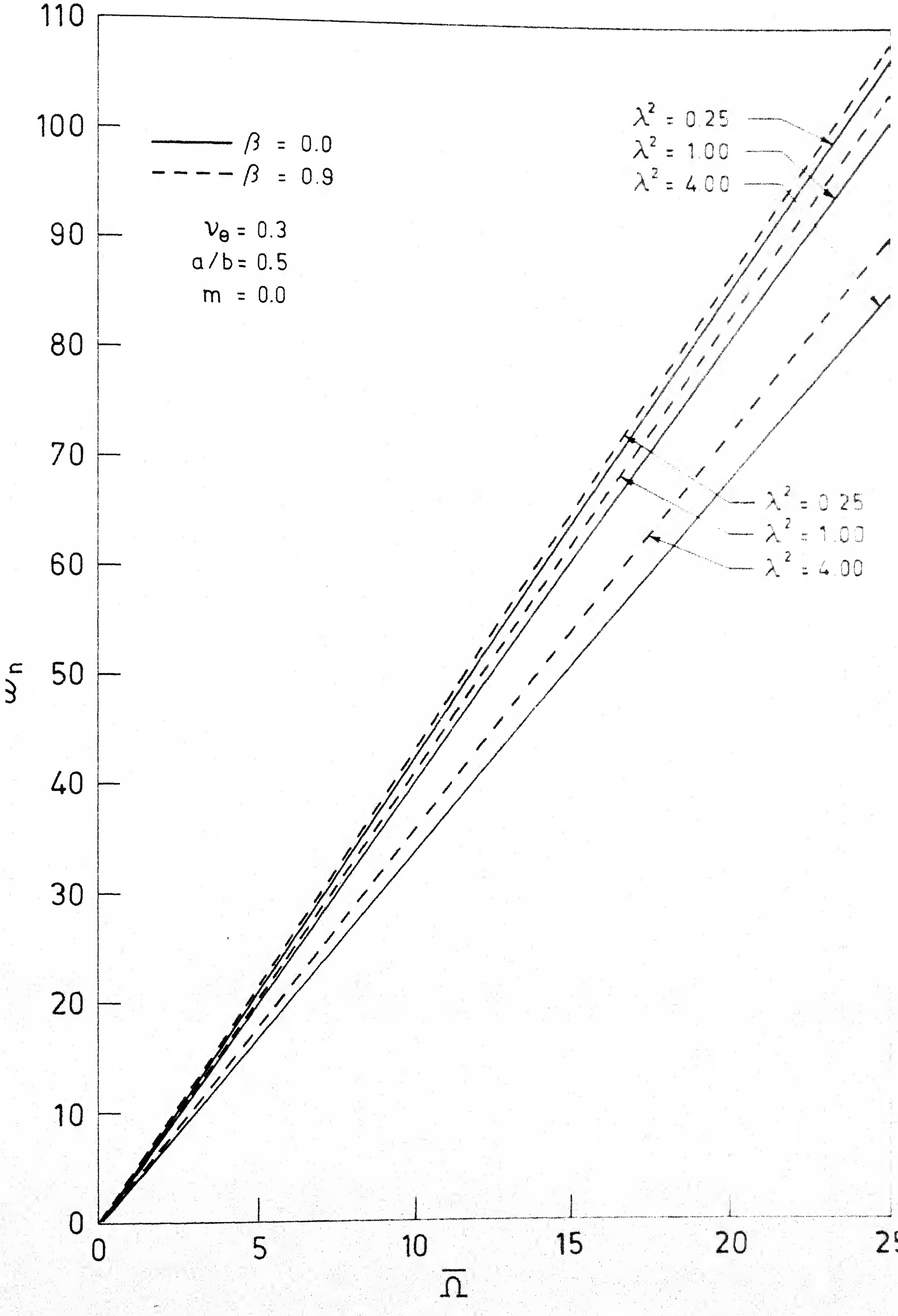


Fig. 17 Variation of frequency with speed of a membrane.

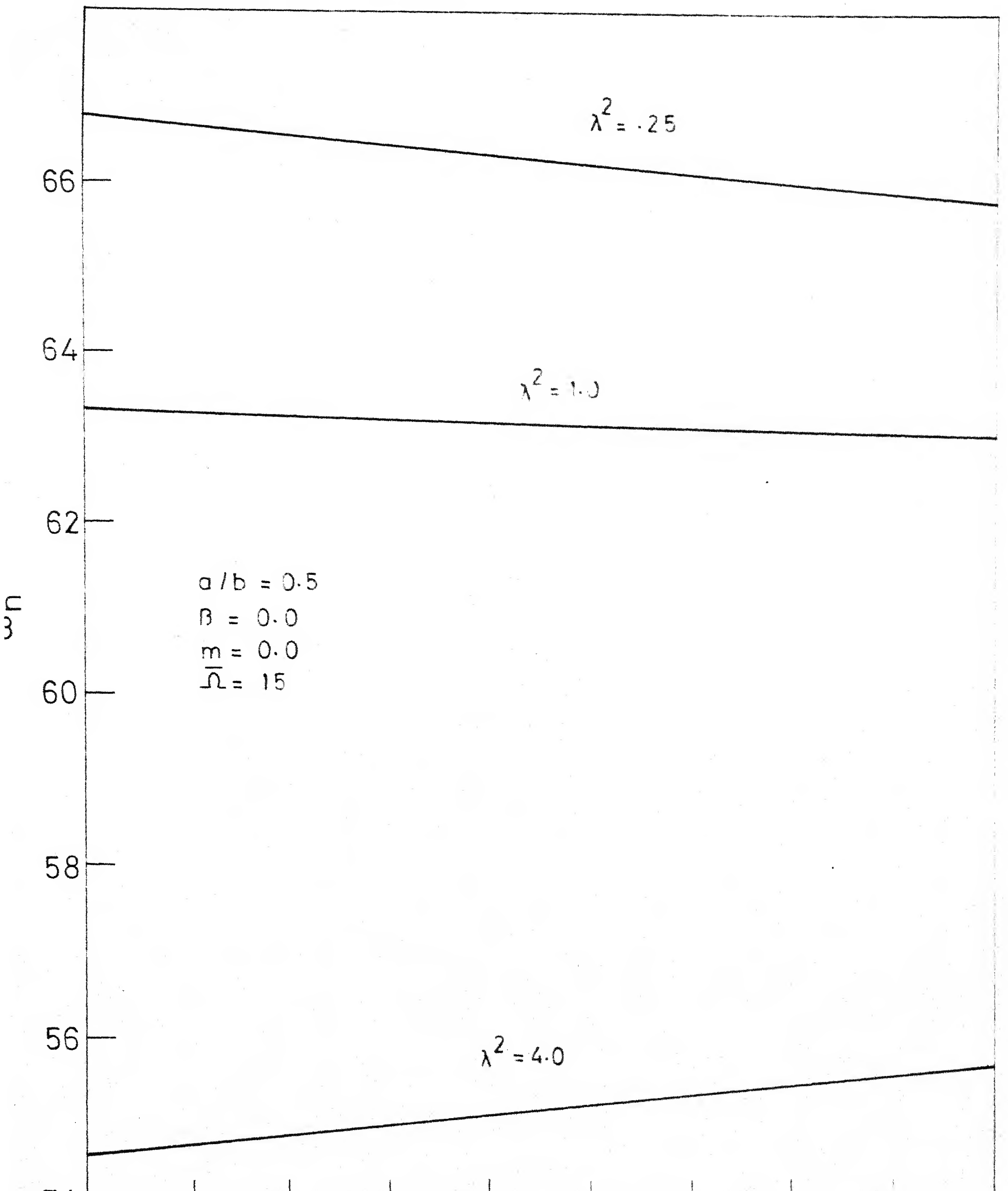


FIG. 18 VARIATION OF FREQUENCY WITH POISSONS - RATIO

APPENDIX

Approximate formula for determination of natural frequency of an orthotropic circular Plate :

Ramaih and Vijay Kumar [22] has given an approximate formula. Natural frequency of an orthotropic plate can be determined if the frequency of the similar isotropic plate is known, from the following relation;

$$\frac{\omega_n^2}{D_r} = \left[1-M \left(1- \frac{D_\theta}{D_r}\right) \right] \frac{\omega_{ni}^2}{D} \quad , \; D_\theta < D_r \qquad (A-1)$$

and

$$\frac{\omega_n^2}{D_\theta} = \left[1- M' \left(1- \frac{D_r}{D_\theta}\right) \right] \frac{\omega_{ni}^2}{D} \quad , \; D_r < D_\theta \qquad (A-2)$$

where, $_n$= circular frequency in radians per second

$_{ni}$= circular frequency in radians per second in isotropic case

and the constants M and M are related by the equation

$$M + M' = 1 \qquad (A-3)$$

The variation of M with clamping radius a/b is given in the following table.

Clamping radius, a/b	Nodal diameter		
	0	1	2
0.1	0.578	-	0.413
0.2	0.436	-	0.242
0.3	0.312	-	0.105
0.4	0.212	0.100	0.036
0.5	0.140	0.075	0.019
0.6	0.080	0.050	0.009
0.7	0.042	0.029	0.005
0.8	0.018	0.012	0.002

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